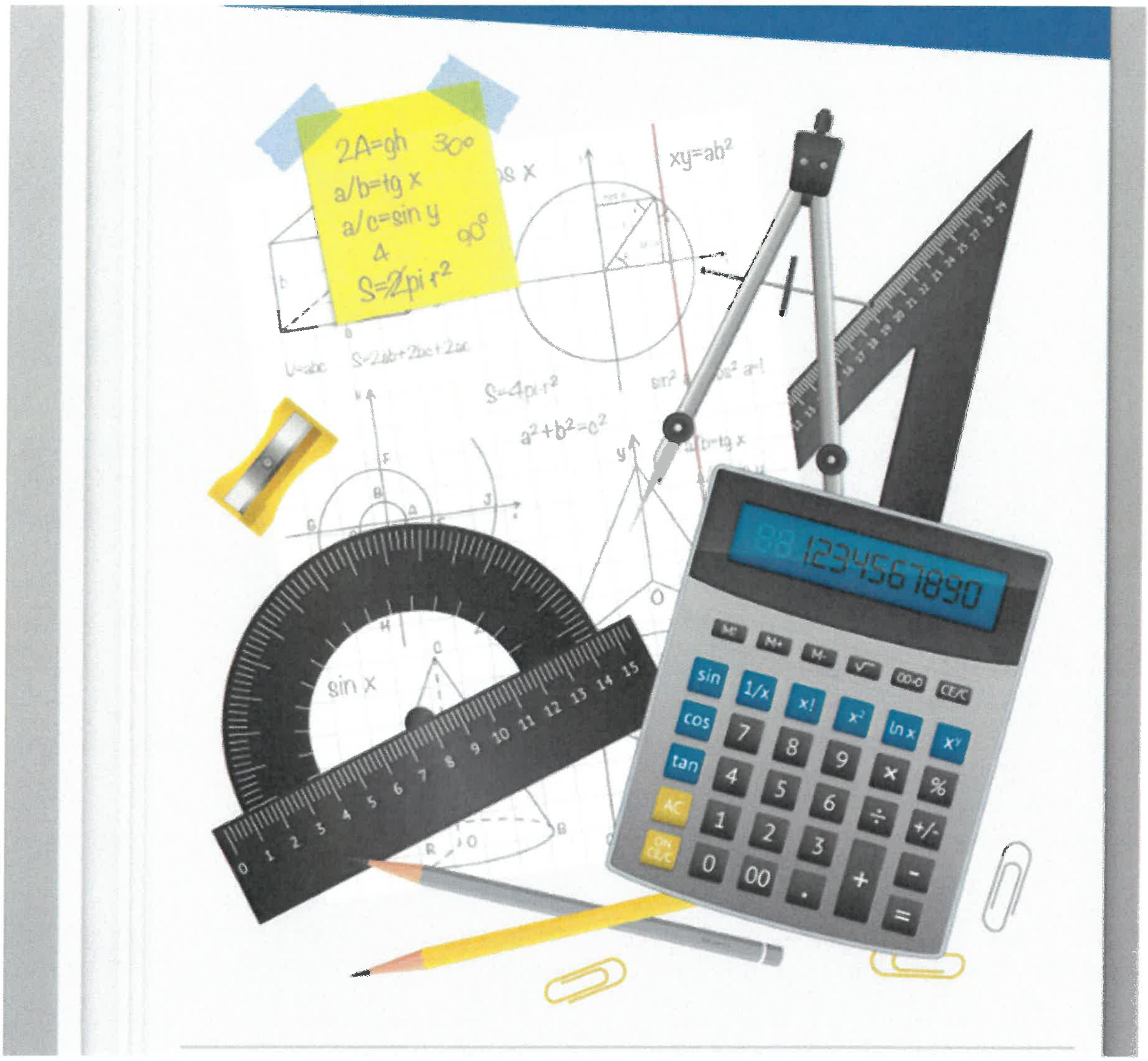
Madani Girls Maths Knowledge Organiser

Key Stage 4



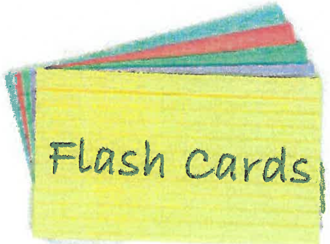
How do we revise with our Knowledge Organisers?

Record It

Record yourself on your phone or tablet reading out the information. These can be listened to as many times as you want!

Teach it!

Teach someone your key facts and the get them to test you, or even test them!



Flash Cards

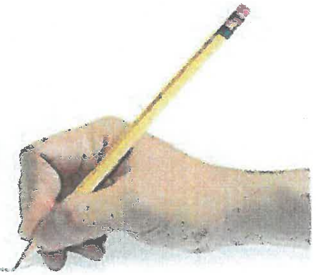
Write the key word or date on one side and the explanation on the other. Test your memory by asking someone to quiz you on either side.

Back to front

Write down the answers and then write out what the questions the teacher may ask to get those answers.

Hide and Seek

Read through your knowledge organiser, put it down and try and write out as much as you can remember. Then keep adding to it until its full!



Sketch it

Draw pictures to represent each of the facts or dates. It could be a simple drawing or something that reminds you of the answer.

Post its

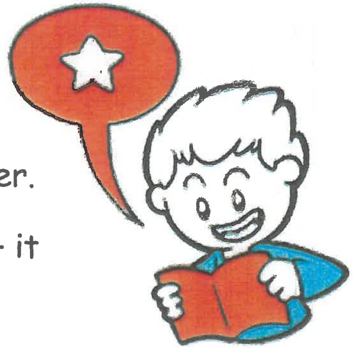
Using a pack of post-it notes, write out as many of the keywords or dates as you can remember in only 1 minute!

Practice!

Some find they remember by simply writing the facts over and over again.

Read Aloud

Simply speak the facts and dates out loud as you're reading the Knowledge Organiser. Even try to act out some of the facts - it really helps you remember!



USING NUMBER...
Non-calculator methods

@whisto_maths

What do I need to be able to do?

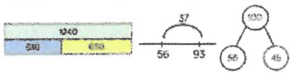
By the end of this unit you should be able to:

- Use mental/written methods for the four number operations
- Use four operations for fractions
- Write exact answers
- Round to decimal places and significant figures
- Estimate solutions
- Understand limits of accuracy
- Understand financial maths

Keywords

Truncate: to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)
Round: making a number simpler, but keeping its place value close the what it originally was
Credit: money that goes into a bank account
Debit: money that leaves a bank account
Profit: the amount of money after income - costs
Tax: money that the government collects based on income, sales and other activities.
Balance: The amount of money in a bank account
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value

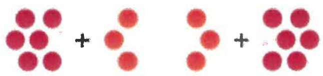
Addition/ Subtraction



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative



6 + 3 = 3 + 6

The order of addition does not change the result

Subtraction the order has to stay the same

360 – 147 = 360 – 100 – 40 – 7

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	4	2	7
–	2	4	9

Remember the place value of each column. You may need to move 10 ones to the ones column to be able to subtract

Decimals have the same methods remember to align the place value

Division methods

3584 ÷ 7 = 512

Short division

$$\begin{array}{r} 5\ 1\ 2 \\ 7 \overline{)\ 3\ {}^{3}5\ 8\ {}^{1}4} \end{array}$$

Complex division

÷ 24 = ÷ 6 ÷ 4

Break up the divisor using factors

Division with decimals

The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

24 ÷ 0.02 ⟶ 24 ÷ 0.2 ⟶ 240 ÷ 2

All give the same solution as represent the same proportion.
Multiply the values in proportion until the divisor becomes an integer

Multiplication methods

Long multiplication (column)

Grid method

Repeated addition

Less effective method especially for bigger multiplication

Multiplication with decimals

Perform multiplications as integers

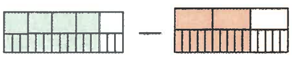
e.g. 0.2 x 0.3 ⟶ 2 x 3

Make **adjustments** to your answer to match the question: 0.2 x 10 = 2
0.3 x 10 = 3

Therefore 6 ÷ 100 = 0.06

Four operations with fractions

Addition and Subtraction



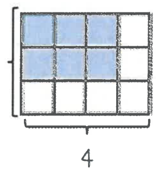
$\frac{4}{5} - \frac{2}{3}$

$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Multiplication

$\frac{3}{4} \times \frac{2}{3}$

$= \frac{6}{12} = \frac{1}{2}$



Division

$\frac{2}{5} \div \frac{3}{4}$

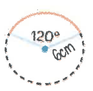
$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

$= \frac{8}{15}$

Exact Values

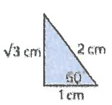
Leave in terms of π



$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi = 12\pi$

Leave as a surd



$\text{Tan}30 = \frac{1}{\sqrt{3}}$

Rounding

2.46192 (to 2dp) - Is this closer to 2.46 or 2.47

2.46 | 192

This shows the number is closer to 2.46

Significant Figures

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00000037 to 1 significant figure is 0.0000004

SF: Round to the first nonzero number

Estimation

Round to 1 significant figure to estimate

21.4 x 3.1 ≈ 20 x 3 ≈ 60

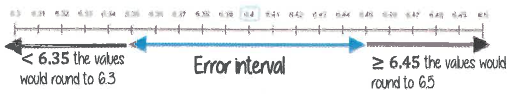
The equal sign changes to show it is an estimation

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

Limits of accuracy

A width **w** has been rounded to 6.4cm correct to 1dp



The error interval

$6.35 \leq w < 6.45$

Any value within these limits would round to 6.4 to 1dp

A width **w** has been truncated to 6.4cm correct to 1dp.

$6.4 \leq w < 6.5$

Any value within these limits would truncate to 6.4 to 1dp

USING NUMBER...

Types of number & sequences

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand factors and multiples
- Express numbers as a product of primes
- Find the HCF and LCM
- Describe and continue sequences
- Explore sequences
- Find the nth term of a linear sequence

Keywords

Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
HCF: highest common factor. The biggest factor that numbers share.
LCM: lowest common multiple. The first multiple numbers share.
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed nonzero number
Sequence: items or numbers put in a pre-decided order

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15...

This list continues and doesn't end

3x, 6x, 9x ...

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5 ← Not an integer

Factors

Arrays can help represent factors

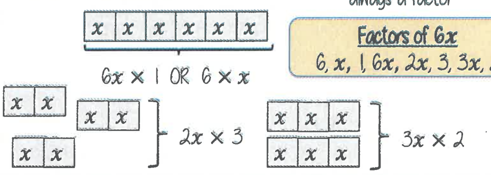
5 x 2 or 2 x 5

Factors of 10: 1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions



Factors of 6x: 6, x, 1, 6x, 2x, 3, 3x, 2

Prime numbers

- Integer
- Only has 2 factors
- and itself

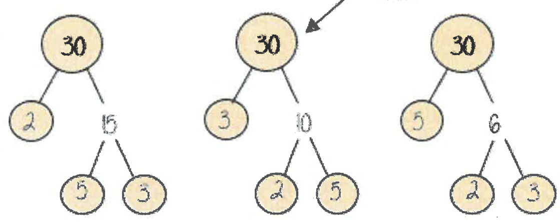
2 — The first prime number. The only even prime number

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

eg 60   30 x 2   2 x 3 x 5 x 2
150   30 x 5   2 x 3 x 5 x 5

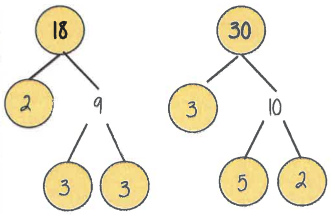
Finding the HCF and LCM

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18
30: 1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share — HCF = 6

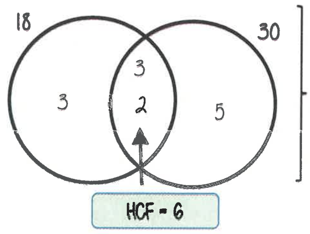


LCM – Lowest common multiple

LCM of 18 and 30

18: 18, 36, 54, 72, 90
30: 30, 60, 90

The first time their multiples match — LCM = 90



LCM = 90

Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found my multiplication/ division between terms.

Term to term rule – how you get from one term (number in the sequence) to the next term

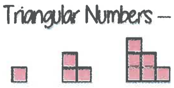
Position to term rule – take the rule and substitute in a position to find a term. Eg. Multiply the position number by 3 and then add 2

Other sequences

Fibonacci Sequence
1, 1, 2, 3, 5, 8 ...

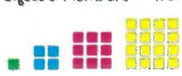
Each term is the sum of the previous two terms

Triangular Numbers – look at the formation



1, 3, 6, 10, 15 ...

Square Numbers – look at the formation



1, 4, 9, 16...

Sequences are the repetition of a patten

Finding the nth term

This is the 4 times table → 4, 8, 12, 16, 20....

4n

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

4n + 3

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

USING NUMBER...

Indices & Roots

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:
- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.
Coefficient: The number used to multiply a variable

Square and cube numbers

R

Square numbers

1, 4, 9, 16...

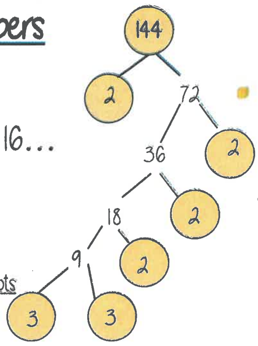
144 = 2 x 2 x 2 x 2 x 3 x 3

(2 x 2 x 3) x (2 x 2 x 3)

12 x 12

Prime factors can find square roots

$\sqrt{144} = 12$



Cube numbers

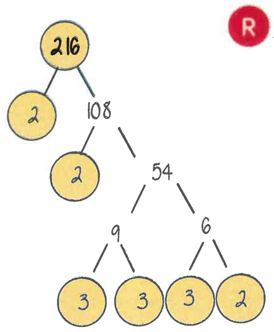
1, 8, 27, 64, 125...

216 = 2 x 2 x 2 x 3 x 3 x 3

(2 x 3) x (2 x 3) x (2 x 3)

6 x 6 x 6

$\sqrt[3]{216} = 6$



Higher powers and roots

x^n — n – power (number of times multiplied by itself)

x – the base number

$\sqrt[n]{x}$ — Finding the nth root of any value

Other mental strategies for square roots

$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$
$= 9 \times 100$
$= 900$

Standard form

R

$A \times 10^n$

Any number between 1 and less than 10 (A)

Any integer (n)

0.001
$1 \times \frac{1}{1000}$
1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers in standard form with negative powers will be less than 1

$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$

Example

3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Addition/ Subtraction Laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

Zero and negative indices

$x^0 = 1$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$

$= a^{6-6} = a^0 = 1$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Powers of powers

$(x^a)^b = x^{ab}$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated Use the addition law for indices

$(2^3)^4 = 2^{12}$ ← $a \times b = 3 \times 4 = 12$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

Standard form calculations

R

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1
$= 600000 + 800000$
$= 1400000$
$= 1.4 \times 10^6$

Method 2
$= (6 + 8) \times 10^5$
$= 14 \times 10^5$ ← This is not the final answer
$= 1.4 \times 10^1 \times 10^5$
$= 1.4 \times 10^6$

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ ← Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$(1.5 \div 0.3) \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

STANDARD FORM

What do I need to be able to do?

You should be able to:

- Write numbers in standard form
- Convert numbers written in standard form to ordinary numbers
- Order numbers in standard form
- Add/subtract numbers in standard form
- Multiply/divide numbers in standard form
- Use a calculator when working with standard form

Key Words

Standard Form: a system of writing very big or small numbers

Commutative: changing the order of operations doesn't change the result

Base: the number that gets multiplied by a power

Power: the number of times the number is used in a multiplication

Index: power (see above)

Exponent: power (see above)

Negative: a value below zero

Converting ordinary numbers into standard form

$A \times 10^n$

Any integer (n)

Any number between 1 and 10 (A)

Examples

700	12500	0.00034
$= 7 \times 10 \times 10$	$= 1.25 \times 10 \times 10 \times 10 \times 10$ (must be between 1 and 10)	Remember a negative power doesn't make the answer negative, just closer to 0!
$= 7 \times 10^2$	$= 1.25 \times 10^4$	$= 3.4 \times 10^{-4}$

Converting standard form into ordinary numbers

Example 1

2×10^3

$= 2 \times 10 \times 10 \times 10$

$= 2000$

Example 2

4.12×10^2

$= 4.12 \times 10 \times 10$

$= 412$

Non-Examples ✗

$12 \times 10^2 = 1200$ (must be between 1 and 10!)

$1.84 \times 10^{0.9} = 14.62$ (must be an integer!)

$6.4 \times 8^3 = 3276.8$ (must be a power of 10!)

Index Laws Recap

$10^3 = 10 \times 10 \times 10 = 1000$

$10^2 = 10 \times 10 = 100$

$10^1 = 10$

$10^0 = 1$

$10^{-1} = \frac{1}{10}$

$10^{-2} = \frac{1}{100} = \frac{1}{10^2}$

$10^{-3} = \frac{1}{1000} = \frac{1}{10^3}$

(+1, ×10 going up; −1, ÷10 going down)

Each time I add one to the power, I multiply by 10

Each time I take one from the power, I divide by 10

Therefore, 10 to the power of 0 is 1. Remember anything to the power 0 is 1

If we carry this on, we can even say what 10 to the power of a negative number is!

We can even spot that 10 to the power of −2 is the same as 1 over 10 to the power of 2 (or 10 squared)

Ordering Numbers in Standard Form

10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

3.1×10^2	4.12×10^4	2×10^{-2}	3.281×10^3	2.4×10^{-2}
310	41200	0.02	3281	0.024

$2 \times 10^{-2} \rightarrow 2.4 \times 10^{-2} \rightarrow 3.1 \times 10^2 \rightarrow 3.281 \times 10^3 \rightarrow 4.12 \times 10^4$

Using a Calculator

If we need to write 1.3×10^3 in our calculator;

You're going to need this button here!

Input 1.3 and then press 

Then press 3 for the power.

Your calculator will often give you the solution to your sum, if it is suitably big/small, in standard form

Adding and Subtracting Numbers in Standard Form

$(2.1 \times 10^6) + (3.3 \times 10^3)$

Foolproof method: convert both numbers to ordinary numbers and then add

$(2.1 \times 10^6) + (3.3 \times 10^3)$

$2,100,000 + 3300$

$= 2,103,300$

$= 2.103 \times 10^6$

You should leave your answer in the form given in the question

$(7.32 \times 10^{-1}) - (2.8 \times 10^{-3})$

$0.732 - 0.0028$

$= 0.7292$

$= 7.292 \times 10^{-1}$

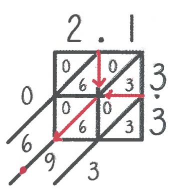
Remember, the best way to work out a subtraction is with column method!

$$\begin{array}{r} 0.7320 \\ -\ 0.0028 \\ \hline 0.7292 \end{array}$$

Multiplying and Dividing Numbers in Standard Form

$(2.1 \times 10^6) \times (3.3 \times 10^3)$

In multiplication and division problems, you can multiply the A values and the look at the powers of 10



$2.1 \times 3.3 \times 10^6 \times 10^3$

$= 6.93 \times 10^6 \times 10^3$

Remember $a^m \times a^n = a^{m+n}$

$= 6.93 \times 10^9$

$(2.8 \times 10^8) \div (7 \times 10^5)$

$\frac{2.8 \times 10^8}{7 \times 10^5} = \frac{0.4 \times 10^8}{10^5} = 0.4 \times 10^3$

Remember $a^m \div a^n = a^{m-n}$

BUT... 0.4×10^3 is not in standard form, as A is not a number between 1 and 10! So...

$0.4 \times 10^3 = 400$

$= 4 \times 10^2$

— PROPORTION...
Percentages and Interest

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:
- Convert and compare FDP
- Work out percentages of amounts
- Increase/ decrease by a given percentage
- Express one number as a percentage
- Calculate simple and compound interest
- Calculate repeated percentage change
- Find the original value
- Solve problems with growth and decay

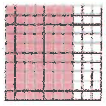
Keywords

Exponent: how many times we use a number in multiplication. It is written as a power
Compound interest: calculating interest on both the amount plus previous interest
Depreciation: a decrease in the value of something over time.
Growth: where a value increases in proportion to its current value such as doubling
Decay: the process of reducing an amount by a consistent percentage rate over time.
Multiplier: the number you are multiplying by
Equivalent: of equal value.

Compare FDP (R)

Comparisons are easier in the same format

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares, 70 'hundredths' = 7 'tenths', 0.7 →



→ 70 hundredths = 70%

Using a calculator: ▪/□ → S⇔D Convert to a decimal → × 100 converts to a percentage

This will give you the answer in the simplest form

Be careful of recurring decimals
e.g. $\frac{1}{3}$ = 0.3333333 = $0.\dot{3}$
The dot above the 3

Fraction/ Percentage of amount (R)

Find $\frac{3}{5}$ of £60

£60: £12 | £12 | £12 | £12 | £12 — £36

Remember $\frac{3}{5}$ = 60%

10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

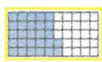
Remember $\frac{3}{5}$ = 60% = 0.6
60% of £60 = 0.6 x 60 = £36

Percentage increase/decrease (R)

100% — 42% — Decrease by 58%
100% - 58% = 42%
100 – 0.58 = 0.42 ← Multiplier Less than 1

100% — 12% — Increase by 12%
100% + 12% = 112%
100 + 0.12 = 1.12 ← Multiplier More than 1

Express as a percentage (R)



27 per every 50 shaded: $\frac{27}{50}$ — 54 per every 100 shaded: $\frac{54}{100}$ — 54%

$\frac{13}{30}$ → ▪/□ → $\frac{13}{30}$ → × 100 → 43.3333...% → 43%

Can't use equivalence easily to find 'per hundred'

Decimal percentages are still a percentage.

Simple and compound interest

Simple Interest

James invests £2000 at 5% simple interest

100% £2000 | 5% £100

The original value increases by this amount every year

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 Original amount: £100
£100 | 10% £10 — Y1: £110
£110 | 10% £11 — Y2: £121
£121 | 10% £12.10 — Y3: £133.10

The multiplier 1.10 repeats each year

Find the original value

Percentage calculations

Original amount × Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test

Original x 0.6 = 24

60% — 24 — Total questions on test

24 ÷ 0.6 = 40 marks
10% = 6
100% = 40

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?

100% | 20% — £3000

Original x 1.2 = 3000
120% = £3000
10% = £250
100% = £2500

Repeated percentage change

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 | x 1.10 | x 1.10 | x 1.10

£100 x 1.10 (3)

Original amount — Repeated multiplier — Number of occurrences

Depreciation

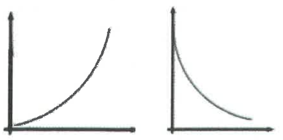
Depreciation calculations use multipliers less than 1

Multipliers are commutative – an overall multiplier effect can be calculated by combining the multipliers separately

e.g. Increase of 10% then a reduction of 10%: x 1.10 | x 0.9 → x 0.99 The multiplier

Growth and decay

Compound growth — Compound decay



Compound growth and compound decay are exponential graphs

Decay – the values get closer to 0
The constant multiplier is less than one

Growth – the values increase exponentially
The constant multiplier is more than one

- PROPORTION...

Ratios and fractions

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare quantities using ratio
- Link ratios and fractions and make comparisons
- Share in a given ratio
- Link Ratio and scales and graphs
- Solve problems with currency conversions
- Solve 'best buy' problems
- Combine ratios

Keywords

Ratio: a statement of how two numbers compare
Equivalent: of equal value
Proportion: a statement that links two ratios
Integer: whole number, can be positive, negative or zero.
Fraction: represents how many parts of a whole.
Denominator: the number below the line on a fraction. The number represent the total number of parts.
Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken
Origin: (0,0) on a graph. The point the two axes cross
Gradient: The steepness of a line

Compare with ratio (R)

"For every dog there are 2 cats"

Dogs: Cats

1:2

The ratio has to be written in the same order as the information is given.

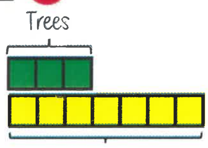
e.g. 2:1 would represent 2 dogs for every 1 cat.

Units have the be of the same value to compare ratios

Ratios and fraction (R)

Trees: Flowers

3 : 7



Ratio

Fraction of trees

$\frac{\text{Number of parts of in group}}{\text{Total number of parts}}$ → $\frac{3}{10}$

Fraction

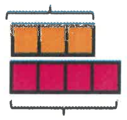
Sharing a whole into a given ratio (R)

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

☐ = one part = £50

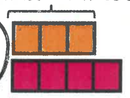
Put back into the question

James: Lucy

×50 3 : 4 ×50

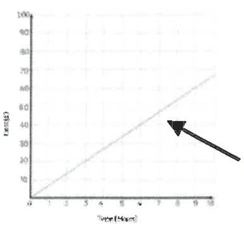
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Ratio and graphs (R)



Graphs with a constant ratio are directly proportional

- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale (R)

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image : Real life
×10 1cm : 30cm ×10
10cm : 300cm

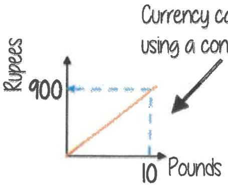
Conversion between currencies (R)

For every £1 I have 90 Rupees

£1 = 90 Rupees ← Currency is directly proportional

×10 £1 = 90 Rupees ×10
£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

×7 £1 = 90 Rupees ×7 ← 630 ÷ 90 = 7
£7 = 630 Rupees

Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

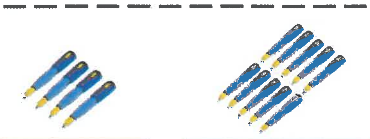
The question states that **this part has to be 1 unit**. Therefore **Divide by 4**

4 : 20
1 : 5

This side has to be divided by 4 too – to keep in **proportion**

the n part does not have to be an integer for this type of question

Best buys



4 pens costs £2.60	10 pens costs £6.00

'1 pen costs...' £2.60 ÷ 4 = **£0.65** £6.00 ÷ 10 = **£0.60**

'1-pound buys...' 4 ÷ 2.60 = **1.54 pens** 10 ÷ 6 = **1.67 pens**

You could work out how much 40 pens are and then compare

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

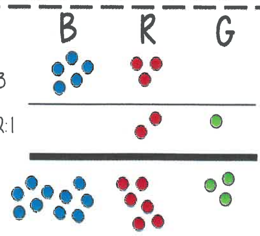
The best value means £1 buys you more pens

Combining ratios

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green



10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

– REASONING WITH ALGEBRA…

Testing conjectures

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

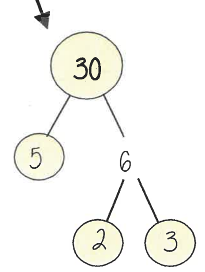
Keywords

Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Prime: an integer with only 2 factors.
HCF: highest common factor (biggest factor two or more numbers share)
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Verify: the process of making sure a solution is correct
Proof: logical mathematical arguments used to show the truth of a statement
Binomial: a polynomial with two terms
Quadratic: a polynomial with four terms (often simplified to three terms)

Factors, Multiples and Primes

R

Multiplication part-whole models



All three prime factor trees represent the same decomposition

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18
30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors are factors two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54
12: 12, 24, 36, 48, 60

Common multiples are multiples two or more numbers share

True or False?

Conjecture

A pattern that is noticed for many cases

1, 2, 4, …
The numbers in the sequence are doubling each time.

Counterexamples

This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

Always, Sometimes, Never true.

Always – Every value always supports the statement

Sometimes – Examples show the statement being true and counter examples to show when it is false.

Never – No example supports the statement

Examples to try
- 0 and 1
- Fractions
- Negative numbers

Show that

Numerical verification – Show the stages to a solution with numerical values

Algebraic verification – Show algebraic properties of the solution. You may want to use pictorial images to support this

Proof – Simple proofs using algebra

Compare the left hand side of an equation with the right hand side – are they the same or different?

Conjectures

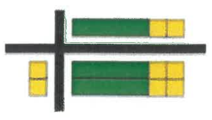
Even $(2n)$ – Multiple of 2

Odd $(2n+1)$ – One more than any even

Use numerical verification first
Use pictorial verification – the representations of numbers of odd and even

Expanding binomials

$2(x+2) \equiv 2x+4$

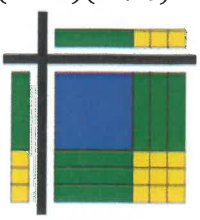


Algebra tiles can represent a binomial expansion
Has two terms

Algebra tiles

Positive values

$(x+3)(x+3) \equiv x^2+6x+9$



This is a quadratic. It has four terms which simplified to three terms

The order of the binomial has no impact on the outcome.
e.g. $(x+3)(3+x)$

Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Positions of numbers in relation to n form expressions.
E.g. one space to the right of n
$n+1$

E.g. One row below n
$n+10$

The size of the grid for generalisation changes the relationship statements

– DEVELOPING ALGEBRA...
Representing solutions of equations and inequalities

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

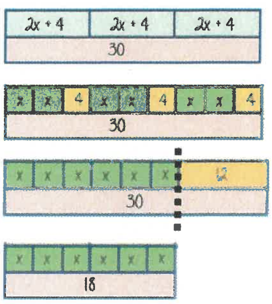
- Form and solve equations and inequalities
- Represent and interpret solutions on a number line as inequalities
- Draw straight line graphs and find solutions to equations
- Form and solve equations and inequalities with unknowns on both sides

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Identity: On equation where both sides have variables that cause the same answer includes ≡
Linear: an equation or function that is the equation of a straight line
Intersection: the point that two lines meet
Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Solve equations (R)

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

-12 -12

$6x = 18$

$\div 6$ $\div 6$

Substitute to check your answer. This could be negative or a fraction or decimal

$x = 3$

Form and solve inequalities (R)

Two more than treble my number is greater than 11

Form $x \rightarrow \times 3 \rightarrow +2 \rightarrow 11$

$3x + 2 > 11$

Solve $x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$

$x > 3$

Solutions on a number line

$x < 1$ $x \leq 1$ $x > 1$ $x \geq 1$

Both represent values less than 1

Includes the value 1

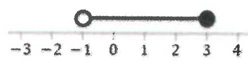
Both represent values more than 1

Includes the value 1

● Includes the value it sits above

○ Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1



$-1 < x \leq 3$

This includes the integer values 0,1,2,3

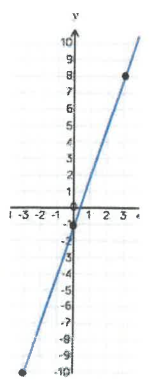
Plotting straight line graphs (R)

$y = 3x - 1$ → 3 × the x coordinate then – 1

x	–3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

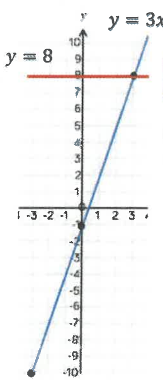
Remember to join the points to make a line

Find solutions graphically

For linear equations there is only one point the graph meets the x value

$x = 2$
$y = 4$

These two lines will cross at (2,4) because they are just x= and y= they are parallel to axes and meet in one place



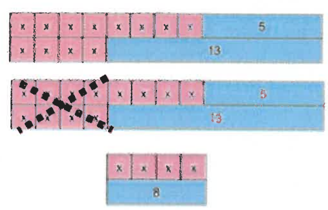
$3x - 1 = 8$

Remember equation of a line format is y = mx + c

The solution is the point the two lines meet (3,8)

Equations: unknown on both sides (R)

$8x + 5 = 4x + 13$



$8x + 5 = 4x + 13$

$-4x$ $-4x$

$4x + 5 = 13$

-5 -5

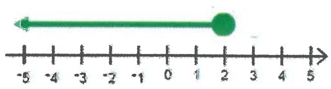
$4x = 8$

$\div 4$ $\div 4$

$x = 2$

Inequalities: unknown on both sides

$8x + 5 \leq 4x + 13$ → $x \leq 2$



Any value 2 or less will satisfy this inequality

– DEVELOPING ALGEBRA...
Simultaneous Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Substitute: replace a variable with a numerical value
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Eliminate: to remove
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Coordinate: a set of values that show an exact position.
Intersection: the point two lines cross or meet.

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

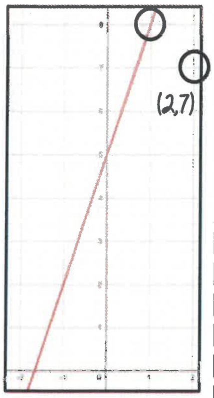
Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5



Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5

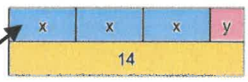
Substituting known variables

A line has the equation 3x + y = 14

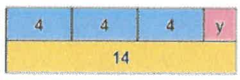
Stephanie knows the point x = 4 lies on that line. Find the value for y.

$x = 4$

$3x + y = 14$



$3(4) + y = 14$



Two different variables, two solutions

$12 + y = 14$
$-12 \quad -12$

$y = 2$

Substituting in an expression

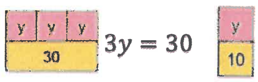
x = 2y

x + y = 30

Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value

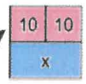
$x = 2y \quad x + y = 30$



$3y = 30$

$3y = 30$
$\div 3 \quad \div 3$
$y = 10$

$x = 2y$



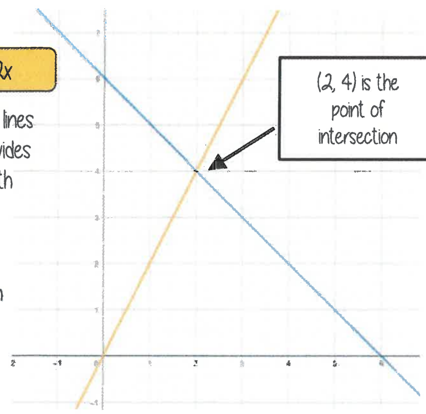
$x = 20$

Solve graphically

x + y = 6

y = 2x

Linear equations are straight lines
The point of intersection provides the x and y solution for both equations

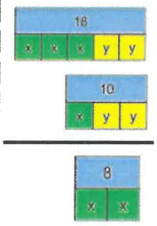


(2, 4) is the point of intersection

The solution that satisfies both equations is

$x = 2$ and $y = 4$

Solve by subtraction

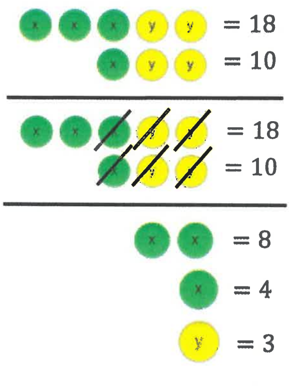


$3x + 2y = 18$
$-\quad x + 2y = 10$

$2x = 8$
$\div 2 \quad \div 2$
$x = 4$

$x + 2y = 10$
$(4) + 2y = 10$
$-4 \quad -4$
$2y = 6$
$\div 2 \quad \div 2$
$y = 3$

$x = 4$
$y = 3$



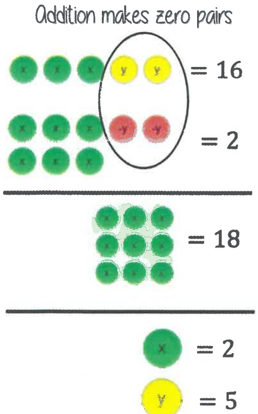
Solve by addition

$3x + 2y = 16$
$+\ 6x - 2y = 2$

$9x = 18$
$\div 9 \quad \div 9$
$x = 2$

$3x + 2y = 16$
$3(2) + 2(y) = 16$
$6 + 2y = 16$
$-6 \quad -6$
$2y = 10$
$y = 5$

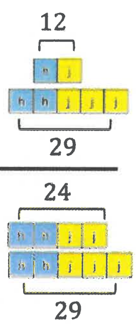
Addition makes zero pairs



Solve by adjusting one

$h + j = 12$
$2h + 2j = 29$

No equivalent values



$2h + 2j = 24$
$2h + 2j = 29$

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

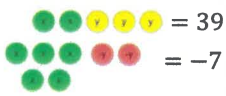
Solve by adjusting both

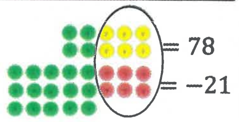
$2x + 3y = 39$
$5x - 2y = -7$

Use LCM to make equivalent x OR y values.
Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$
$15x - 6y = -21$

Now solve by addition





Addition makes zero pairs

– REASONING WITH ALGEBRA...
Straight Line Graphs

@whisto_maths

What do I need to be able to do?

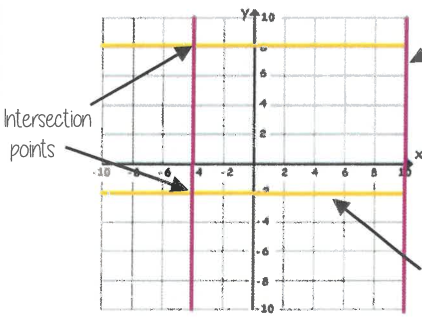
By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use y= mx + c
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line
Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.
Parallel: two lines that never meet with the same gradient.
Co-ordinate: a set of values that show an exact position on a graph.
Linear: linear graphs (straight line) — linear common difference by addition/ subtraction
Asymptote: a straight line that a graph will never meet.
Reciprocal: a pair of numbers that multiply together to give 1.
Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes (R)



'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

e.g (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

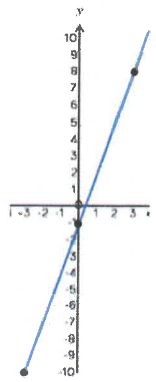
Plotting y = mx + c graphs (R)

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

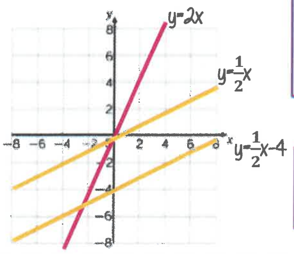
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

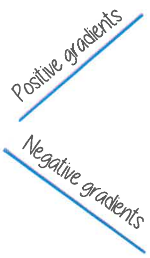
$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



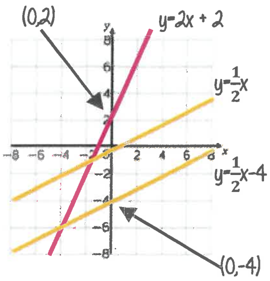
The **greater** the gradient – the **steeper** the line

Parallel lines have the **same** gradient



Compare Intercepts

$y = mx + c$ ← The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$ ← The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

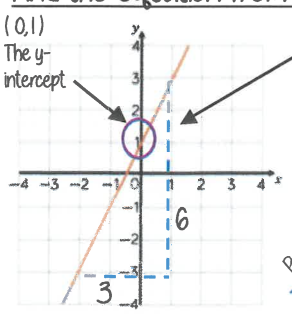
The equation of a line can be rearranged: Eg

y = c + mx

c = y – mx

Identify which coefficient you are identifying or comparing

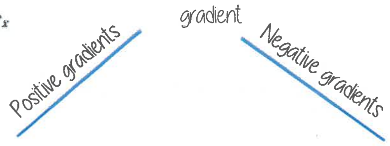
Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient



Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin.

When you have 0 pens this has 0 cost. The gradient shows the price per pen

A box of pens costs £2.30
Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

— PROPORTION...

@whisto_maths

Probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Add, Subtract and multiply fractions
- Find probabilities using likely outcomes
- Use probability that sums to 1
- Estimate probabilities
- Use Venn diagrams and frequency trees
- Use sample space diagrams
- Calculate probability for independent events
- Use tree diagrams

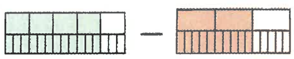
Keywords

Event: one or more outcomes from an experiment
Outcome: the result of an experiment.
Intersection: elements (parts) that are common to both sets
Union: the combination of elements in two sets.
Expected Value: the value/ outcome that a prediction would suggest you will get
Universal Set: the set that has all the elements
Systematic: ordering values or outcomes with a strategy and sequence
Product: the answer when two or more values are multiplied together.

Add, Subtract and multiply fractions (R)

Addition and Subtraction

$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Use equivalent fractions to find a common multiple for both denominators

Multiplication

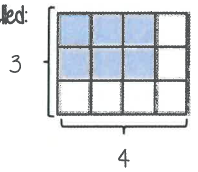
$\frac{3}{4} \times \frac{2}{3}$

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

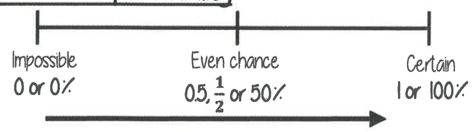
Parts shaded

Total number of parts in the diagram

Modelled:



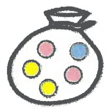
Likeliness of a probability (R)



The more likely an event the further up the probability it will be in comparison to another event. (It will have a probability closer to 1)

Sum to 1 (R)

Probability is always a value between 0 and 1



The probability of getting a blue ball is $\frac{1}{5}$

∴ The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

Experimental data

Theoretical probability — What we expect to happen

Experimental probability — What actually happens when we try it out

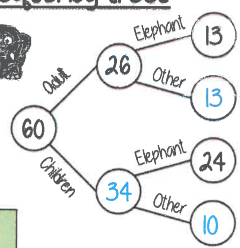
The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.
Theoretical probability is proportional

Tables, Venn diagrams, Frequency trees

Frequency trees

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

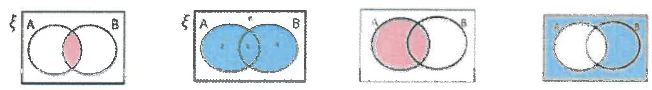
P(adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

Two-way table

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Venn diagram



in set A AND set B
$P(A \cap B)$

in set A OR set B
$P(A \cup B)$

in set A
$P(A)$

NOT in set A
$P(A')$

Sample space (R)

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

P (Even number and tails) = $\frac{3}{12}$

Independent events

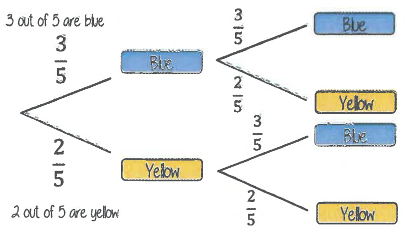
The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

$P(A \text{ and } B) = P(A) \times P(B)$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick.

Because they are replaced the second pick has the same probability



$= P(BB)$ $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

$= P(BY)$ $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

$= P(YB)$ $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

$= P(YY)$ $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

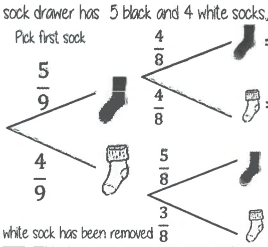
Sum of probabilities = 1

Dependent events

The outcome of the first event has an impact on the second event

Tree diagram for dependent event

A sock drawer has 5 black and 4 white socks. Jamie picks 2 socks from the drawer.



$= P(BB)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P(BW)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P(WB)$ $\frac{4}{9} \times \frac{5}{8} = \frac{20}{72}$

$= P(WW)$ $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$

Sum of probabilities = 1

NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced ∴ the denominator is also reduced for the second pick

— DELVING INTO DATA...

Collecting, representing and interpreting

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What do I need to be able to do?

By the end of this unit you should be able to:

- Construct and interpret frequency tables and polygon. two-way tables, line, bar, & pie charts
- Find and interpret averages from a list and a table
- Construct and interpret time series graphs, stem and leaf diagrams and scatter graphs

Keywords

Population: the whole group that is being studied
Sample: a selection taken from the population that will let you find out information about the larger group
Representative: a sample group that accurately represents the population
Random sample: a group completely chosen by change. No predictability to who it will include.
Bias: a built-in error that makes all values wrong by a certain amount
Primary data: data collected from an original source for a purpose.
Secondary data: data taken from an external location. Not collected directly.
Outlier: a value that stands apart from the data set

Stem and leaf

A way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket

0	7 9
1	4 5 6 8 8
2	1 3
3	0

Key: 1|4 Means 14 years old

Stem and leaf diagrams:
Must include a key to explain what it represents
The information in the diagram should be ordered

Back to back stem and leaf diagrams

Girls		Boys
5	14	
7, 5, 5, 5, 4	15	3, 8, 9
8, 4, 2, 1, 0	16	2, 5, 7, 7, 7, 8, 8, 9
9, 8, 7, 6, 6, 4, 2, 1, 1, 0, 0	17	0, 2, 3, 6, 6, 7, 7
	18	0, 1, 4, 5

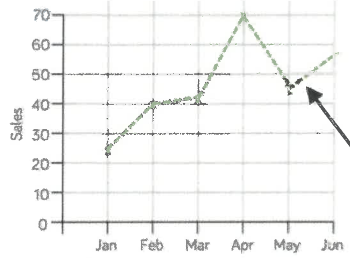
15 | 3. Means 153 cm tall

Back to back stem and leaf diagrams
Allow comparisons of similar groups
Allow representations of two sets of data

Time-Series

R

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

Mean, mode, median – allows for a comparison about more or less average
Range – allows for a comparison about reliability and consistency of data

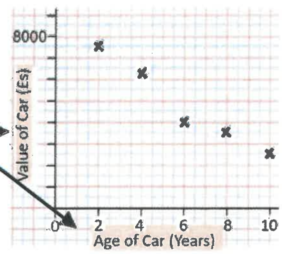
Draw and interpret a scatter graph

R

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

All axes should be labelled



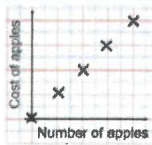
"This scatter graph show as the age of a car increases the value decreases"

The link between the data can be explained verbally

The axis should fit all the values on and be equally spread out

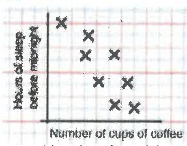
Linear Correlation

R



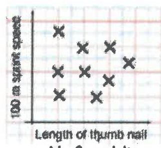
Positive Correlation

As one variable increases so does the other variable



Negative Correlation

As one variable increases the other variable decreases



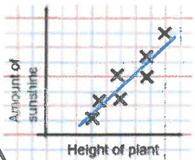
No Correlation

There is no relationship between the two variables

The line of best fit

R

The Line of best fit is used to make estimates about the information in your scatter graph



Things to know:

- The line of best fit DOES NOT need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph

It is only an estimate because the line is designed to be an average representation of the data

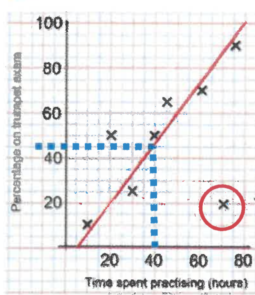
It is always a straight line.

Using a line of best fit

R

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45.



Extrapolation is where we use our line of best fit to predict information outside of our data
This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an 'outlier'
It is an outlier because it doesn't fit this model and stands apart from the data

– DELVING INTO DATA...

Collecting, representing and interpreting

@whisto_maths

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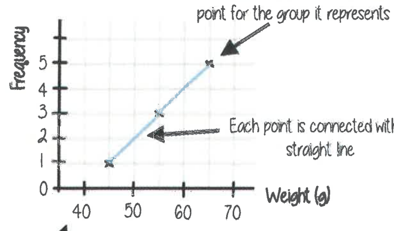
Frequency tables and polygons

x Weight(g)	Frequency
$40 < x \leq 50$	1
$50 < x \leq 60$	3
$60 < x \leq 70$	5

We do not know from grouped data where each value is placed so have to use an estimate for calculations



Each point is plotted at them mid point for the group it represents

Each point is connected with a straight line

MID POINTS

Mid-points are used as estimated values for grouped data. The middle of each group

The data about weight starts at 40. So the axis can start at 40

Mid-point = $\frac{\text{Start point + End point}}{2}$

Two way tables (R)

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.

Extract information to input to the two-way table.

Subgroups each have their own heading

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Needs subgroup totals

Overall total

Draw and interpret Pie Charts (R)

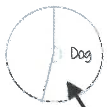
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192°$



Use a protractor to draw. This is 192°

Multiple method

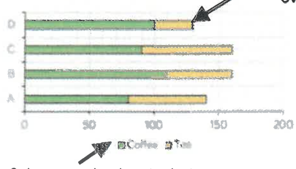
As 60 goes into 360 – 6 times.

Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Comparing Pie Charts: You NEED the overall frequency to make any comparisons

Bar and line charts

Composite bar charts

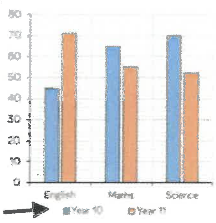


Compare the bars green compared to yellow. The size of each bar is the frequency. Overall total easily comparable.

Categories clearly indicated

Dual bar charts

Bars are compared side by side. Easier to compare subgroups



Categories clearly indicated

Averages from a table (R)

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency	Mid Point	MP x Freq
$40 < x \leq 50$	1	45	45
$50 < x \leq 60$	3	65	195
$60 < x \leq 70$	5	65	325

Overall Frequency: 9

Overall Total : 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

Averages from lists (R)

The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values)

55

Divide the overall total by how many pieces of data you have

55 ÷ 5

Mean = 11

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

This can still be easier if it the data is ordered first

Mode = 8

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8

Put the data in order 4, 8, 8, 11, 24

Find the value in the middle 4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

For Grouped Data

The modal group – which group has the highest frequency

– GEOMETRY...

@whisto_maths

Angles and bearings

What do I need to be able to do?

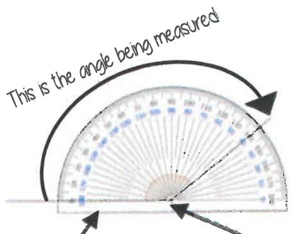
By the end of this unit you should be able to:

- Understand and represent bearings
- Measure and read bearings
- Make scale drawings using bearings
- Calculate bearings using angle rules
- Solve bearings problems using Pythagoras and trigonometry

Keywords

Cardinal directions: the directions of North, South, East, West
Angle: the amount of turn between two lines around their common point
Bearing: the angle in degrees measured clockwise from North.
Perpendicular: where two lines meet at 90°
Parallel: straight lines always the same distance apart and never touch. They have the same gradient.
Clockwise: moving in the direction of the hands on a clock.
Construct: to draw accurately using a compass, protractor and or ruler or straight edge.
Scale: the ratio of the length of a drawing to the length of the real thing
Protractor: an instrument used in measuring or drawing angles.

Measure angles to 180° (R)



Read from 0° on the base line. Remember to use estimation. This is an obtuse angle so between 90° and 180°

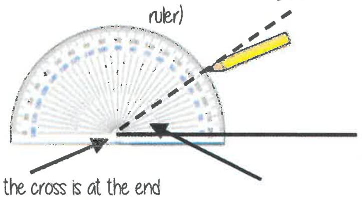
The base line follows the line segment

Make sure the cross is at the point the two lines meet

Draw angles up to 180° (R)

Draw a 35° angle

Make a mark at 35° with a pencil and join to the angle point (use a ruler)



Make sure the cross is at the end of the line (where you want the angle)

The angle

Angle notation (R)

The letter in the middle is the angle
The arc represents the part of the angle

Angle Notation: three letters $A\hat{B}C$ This is the angle at B = 113°
$\angle ABC$ is also used to represent the angle at B.

Scale drawings (R)

1 : 20

For every 1cm on the model there are 20cm in real life

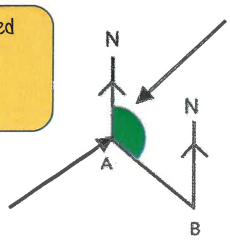
Remember: Scale drawings ONLY change lengths and distances. Angles remain the same

Understand and represent bearings

- A bearing is always measured from NORTH
- It is always given as three figures

The angle indicated starts from the North line at A and joins the path connecting A to B.

This angle shows the bearing of B from A

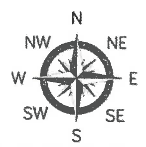


The bearing of B from A is calculated by measuring the highlighted angle

The sentence... "Bearing of ____ from _____" is really important in identifying the bearing being represented

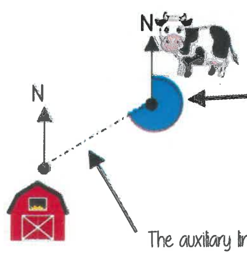
Using estimation it is clear this angle is between 090° and 180°

Directions



Measure and read bearings

The bearing of the cow to the barn

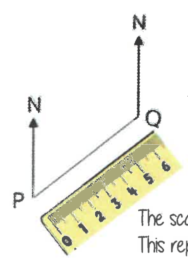


This angle is measured from NORTH
It is measured in a clockwise direction
Estimation indicates this angle is between 180° and 270°
Use a protractor to measure accurately
Remember: bearings are written as three figures.

The auxiliary line is drawn to help you measure and draw the angle that is measured to represent the bearing

Scale drawings using bearings

Remember – angles DO NOT change size in scaled drawings



The bearing measurements do not change from "real life" to images

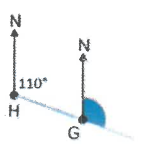
The units in the ratio scale are the same

The scale may need to be calculated from the image.
This represents 30km from P to Q

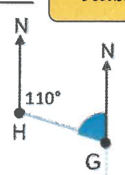
6cm = 30km
6:3,000,000

Bearings with angle rules

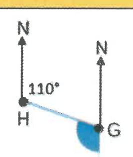
Because two North lines are PARALLEL....



They form corresponding angles and therefore are the same size



They form co-interior angles and add up to 180°

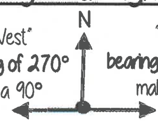


They form alternate angles and therefore are the same size

Bearings with right-angled geometry

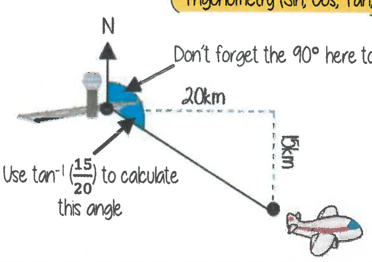
Look for Right-angles
Pythagoras
Trigonometry (Sin, Cos, Tan)

"Due West" **bearing of 270°** makes a 90° angle



"Due East" **bearing of 090°** makes a 90° angle

A plane flies East for 20km then turns South for 15km. Find the bearing of the plane from where it took off.



Don't forget the 90° here too

Use $\tan^{-1}\left(\frac{15}{20}\right)$ to calculate this angle

– CONSTRUCTING IN 2D/3D...
Constructions & congruency

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What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

Keywords

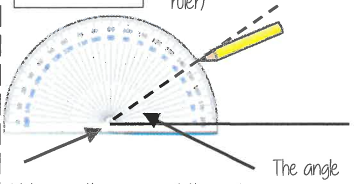
Protractor: piece of equipment used to measure and draw angles
Locus: set of points with a common property
Equidistant: the same distance
Discorectangle: (a stadium) — a rectangle with semi circles at either end
Perpendicular: lines that meet at 90°
Arc: part of a curve
Bisector: a line that divides something into two equal parts
Congruent: the same shape and size

Draw and measure angles

R

Draw a 35° angle

Make a mark at 35° with a pencil And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

R

A picture of a car is drawn with a scale of 1:30

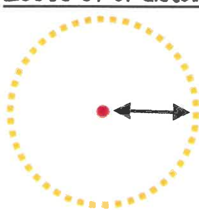
For every 1cm on my image is 30cm in real life

The car image is 10cm

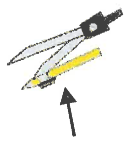
Image	:	Real life
1cm	:	30cm
10cm	:	300cm

× 10 (both sides)

Locus of a distance from a point



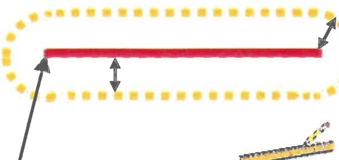
All points are equidistant (the same distance) from the fixed point in the middle.



Equipment needed
The radius is the distance from the fixed point

If the point is in the corner it can only make a quarter circle

Locus of a distance from a straight line

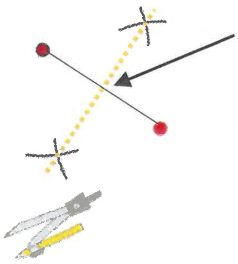


All points are equidistant (the same distance) from line

The ends of the line are fixed points

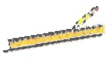
Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points



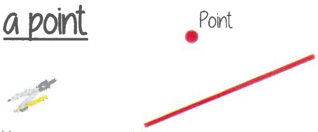
Also a perpendicular bisector
Because if the points are joined, this new line intersects it at a 90°

Keep the compass the same size and draw two arcs from each point



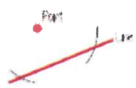
Join the intersections with a ruler.
All points on this line are equidistant from both points

Construct a perpendicular from a point

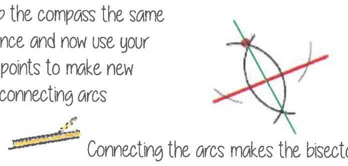


Point
Line

Use a compass and draw an arc that cuts the line. Use the point to place the compass



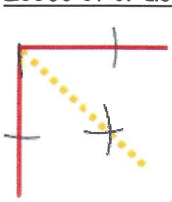
Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



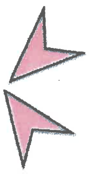
Also an angle bisector
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

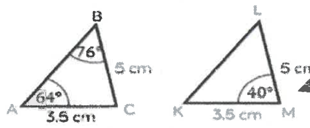
Join the vertex to the intersection

Congruent figures



Congruent figures are identical in size and shape — they can be reflections or rotations of each other

Congruent shapes are identical — all corresponding sides and angles are the same size



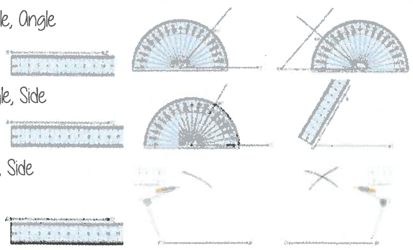
$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

Constructing Triangles

Link to steps → R

Side, Angle, Angle



Side, Angle, Side

Side, Side, Side

Congruent triangles

Side-side-side
All three sides on the triangle are the same size

Angle-side-angle
Two angles and the side connecting them are equal in two triangles

Side-angle-side
Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side
The triangles both have a right angle, the hypotenuse and one side are the same

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— SIMILARITY...

Congruence, similarity & enlargement

What do I need to be able to do?

By the end of this unit you should be able to:

- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Centre of enlargement: the point the shape is enlarged from
Similar: when one shape can become another with a reflection, rotation, enlargement or translation.
Congruent: the same size and shape
Corresponding: items that appear in the same place in two similar situations
Parallel: straight lines that never meet (equal gradients)

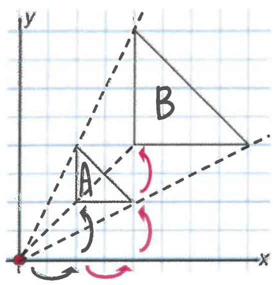
Positive scale factors (R)

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

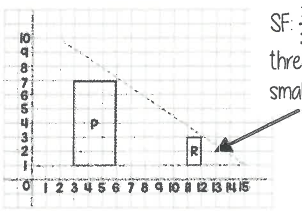
The distance from the point enlarges by 2



Fractional scale factors (R)

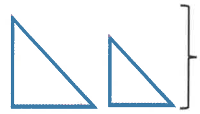
Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15, 1)



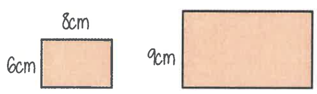
SF: $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180°

Similar shapes

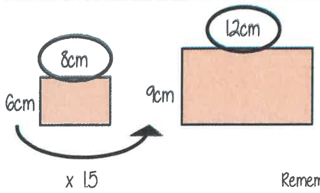


Scale Factor: Both sides on the bigger shape are 1.5 times bigger

Compare sides: 6 : 9 = 2 : 3 8 : 12 = 2 : 3

Both sets of sides are in the same ratio

Information in similar shapes



Compare the equivalent side on both shapes

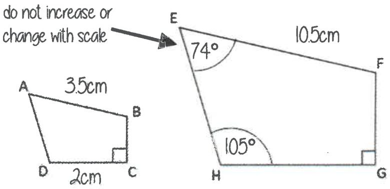
Scale Factor is the multiplicative relationship between the two lengths

Remember angles do not increase or change with scale

Shape ABCD and EFGH are similar

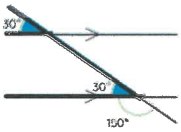
Notation helps us find the corresponding sides

AB and EF are corresponding



Angles in parallel lines (R)

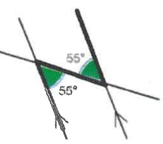
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

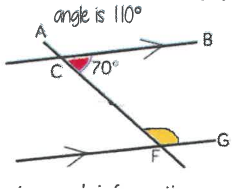
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°



As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

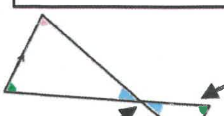
Similar triangles

Shares a vertex

Because corresponding angles are equal the highlighted angles are the same size

Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

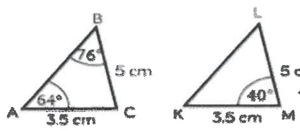


All the angles in both triangles are the same and so similar

Vertically opposite angles

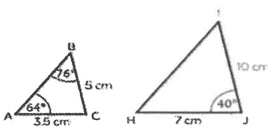
Congruence and Similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

— GEOMETRY...

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Working with circles

What do I need to be able to do?

By the end of this unit you should be able to:

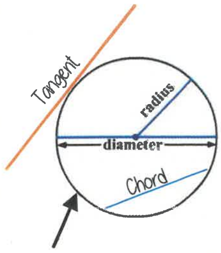
- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere.
- Understand and use surface area of a cone, cylinder and sphere.

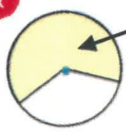
Keywords

Circumference: the length around the outside of the circle – the perimeter
Area: the size of the 2D surface
Diameter: the distance from one side of a circle to another through the centre
Radius: the distance from the centre to the circumference of the circle
Tangent: a straight line that touches the circumference of a circle
Chord: a line segment connecting two points on the curve
Frustrum: a pyramid or cone with the top cut off
Hemisphere: half a sphere
Surface area: the total area of the surface of a 3D shape.

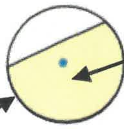
Parts of a circle

R





Sector (part of the circle made from two radii)



Segment (part of the circle made from a chord)

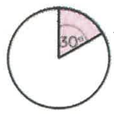
Circumference

An arc is a part of the circumference

Fractional parts of a circle

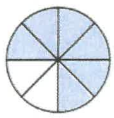
A circle is made up of 360°

Formula to remember
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$\frac{30}{360} = \frac{1}{12}$



$\frac{270}{360}$ of a full circle (in degrees)

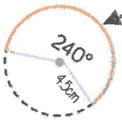
$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is $\frac{\theta}{360}$

θ represents the degrees in the sector

Arc length



Remember an arc is part of the circumference
Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$

Arc length = $\frac{\theta}{360} \times$ circumference

$= \frac{240}{360} \times 9\pi$

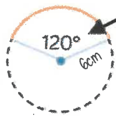
$= \frac{2}{3} \times 9\pi = \underline{6\pi}$

Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360} \times$ circumference $+ 2r$ $\quad = \underline{6\pi + 9}$

Sector area



Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$

Sector area = $\frac{\theta}{360} \times$ area of circle

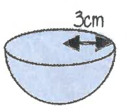
$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi = \underline{12\pi}$

Volume of a sphere

Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a **cubed** value



Volume Sphere = $\frac{4}{3}\pi r^3$

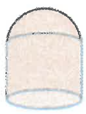
$= \frac{4}{3} \times \pi \times 3^3$

$= \frac{4}{3} \times \pi \times 27 = \underline{36\pi}$

A hemisphere is half the volume of the overall sphere $= 36\pi \div 2$

$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g. cones and cylinders



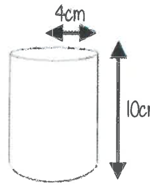
Volume of a cone and a cylinder

Volume Cylinder = $\pi r^2 h$

A cylinder is a prism – cross section is a circle

Volume Cone = $\frac{1}{3}\pi r^2 h$

A cone is a pyramid with a circular base



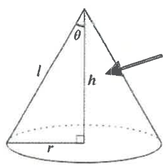
$V = \pi r^2 h$

$= \pi \times 4^2 \times 10$

$= \pi \times 160$

$= \underline{160\pi}$ cm²

Give your answer in terms of π' means NOT in terms of pi $\quad = \underline{502.7}$cm²

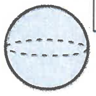


The height of a cone is the perpendicular height from the vertex to the base

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Surface area of a sphere

Surface area = $4\pi r^2$



Radius = 5cm

Surface area = $4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

The curved surface area of a sphere $= \underline{100\pi}$

A hemisphere has the curved surface AND a flat circular face

$= 100\pi \div 2 = 50\pi$

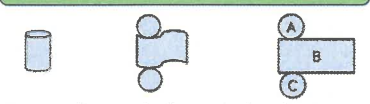
$= 50\pi + \pi \times 5^2$

Hemisphere $= \underline{75\pi}$

Surface area of cones and cylinders

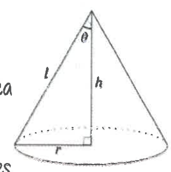
Surface area cylinder = $2\pi r^2 + \pi dh$

Curved surface area Cone = $\pi r l$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles



Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)

Vectors

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What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

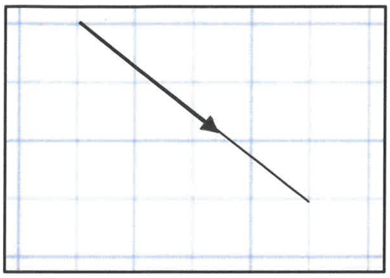
Keywords

Direction: the line our course something is going
Magnitude: the magnitude of a vector is its length
Scalar: a single number used to represent the multiplier when working with vectors
Column vector: a matrix of one column describing the movement from a point
Resultant: the vector that is the sum of two or more other vectors
Parallel: straight lines that never meet

Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another

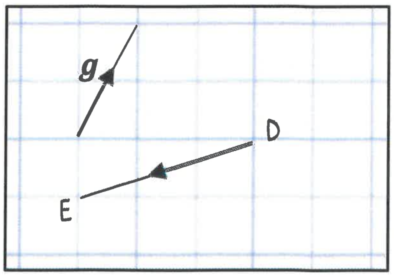
Movement along the x-axis → $\begin{pmatrix}4\\-3\end{pmatrix}$ ← Movement along the y-axis



Vectors show both direction and magnitude

The arrow is pointing in the direction from starting point to end point of the vector.	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E.

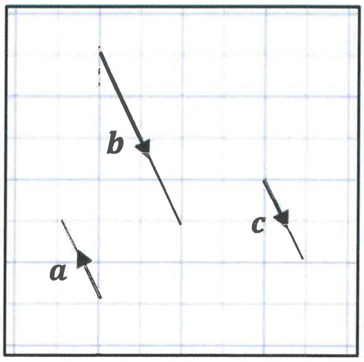
$\overrightarrow{DE} = \begin{pmatrix}-3\\-1\end{pmatrix}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so **g** represents the vector $\boldsymbol{g} = \begin{pmatrix}1\\2\end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$

Multiply **c** by 2 this becomes **b**. The two lines are parallel

$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$

The vectors **a** and **c** are also parallel. A negative scalar causes the vector to reverse direction.

$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$

$\boldsymbol{a} = \begin{pmatrix}-1\\2\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}2\\-4\end{pmatrix}$ $\boldsymbol{c} = \begin{pmatrix}1\\-2\end{pmatrix}$

Addition of vectors

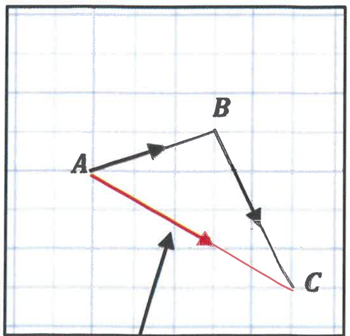
$\overrightarrow{AB} = \begin{pmatrix}3\\1\end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix}2\\-4\end{pmatrix}$

$$\overrightarrow{AB} + \overrightarrow{BC}$$

$$= \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}2\\-4\end{pmatrix}$$

$$= \begin{pmatrix}3+2\\1+-4\end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$$



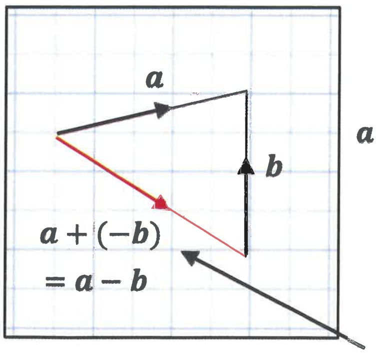
The resultant

Look how this addition compares to the vector $\overrightarrow{AC}$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$$

Addition and subtraction of vectors

$\boldsymbol{a} = \begin{pmatrix}5\\1\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}0\\4\end{pmatrix}$



$$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix}5+&-0\\1+&-4\end{pmatrix} = \begin{pmatrix}5\\-4\end{pmatrix}$$

The resultant is **a** − **b** because the vector is in the opposite direction to b which needs a scalar of -1

- SIMILARITY...

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Trigonometry

What do I need to be able to do?

By the end of this unit you should be able to:

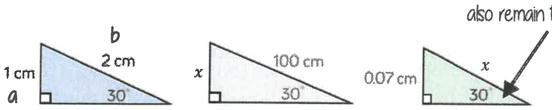
- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Constant: a value that remains the same
Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.
Inverse: function that has the opposite effect.
Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

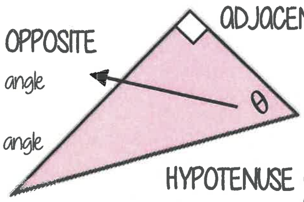
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE
Always opposite an acute angle
Useful to label second
Position depend upon the angle in use for the question

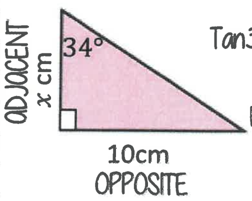
ADJACENT
Next to the angle in question
Often labelled last

HYPOTENUSE
Always the longest side
Always opposite the right angle
Useful to label this first

Tangent ratio: side lengths

$Tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



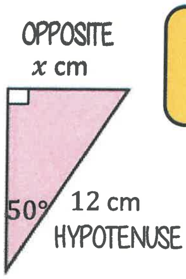
$Tan34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times Tan34 = 10$

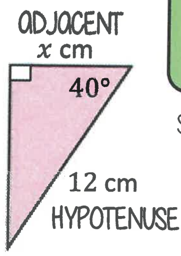
$x = \frac{10}{Tan34} = 14.8cm$

Sin and Cos ratio: side lengths



$Sin\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio

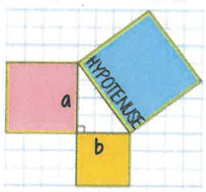


$Cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem (R)



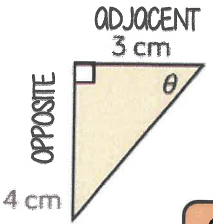
$Hypotenuse^2 = a^2 + b^2$

This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\theta = Tan^{-1} \frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = Sin^{-1} \frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = Cos^{-1} \frac{\text{adjacent side}}{\text{hypotenuse side}}$

$Tan\theta = \frac{3}{4}$

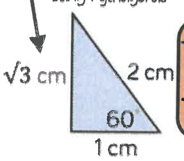
$\theta = Tan^{-1} \frac{3}{4}$

$\theta = 36.9°$

Key angles

This side could be calculated using Pythagoras

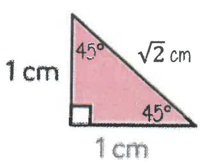
Because trig ratios remain the same for similar shapes you can generalise from the following statements



$Tan30 = \frac{1}{\sqrt{3}}$
$Tan60 = \sqrt{3}$

$Cos30 = \frac{\sqrt{3}}{2}$
$Cos60 = \frac{1}{2}$

$Sin30 = \frac{1}{2}$
$Sin60 = \frac{\sqrt{3}}{2}$



$Tan45 = 1$

$Cos45 = \frac{1}{\sqrt{2}}$

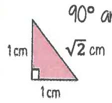
$Sin45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$Tan0 = 0$

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$Sin0 = 0$

$Sin90 = 1$

$Cos0 = 1$

$Cos90 = 0$

Unit 13b: Further Trigonometry

Trigonometry with bounds

The upper bound of a multiplication is always the two upper bounds multiplied together

The lower bound of a multiplication is always the two lower bounds multiplied together

The upper bound of a fraction is always

$$\frac{\textit{Upper bound of the numerator}}{\textit{Lower Bound of the denominator}}$$

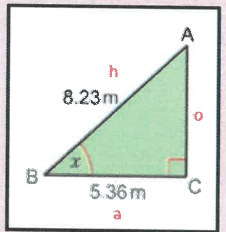
The lower bound of a fraction is always

$$\frac{\textit{Lower bound of the numerator}}{\textit{Upper Bound of the denominator}}$$

In this diagram, the measurements are correct to 3 significant figures.

a Find the upper and lower bounds for the value of x, to 3 decimal places.

b Give the value of x to a suitable level of accuracy.



Steps:

1) Find the upper bound And lower bound of the sides

2) Find the upper and lower Value of x using trigonometry

3) Round both values to the Nearest degree to find a Good estimate for x

UB of h = 8.235
LB of h = 8.225

UB of a = 5.365
LB of a = 5.355

$cos x_{ub} = \frac{5.365}{8.225}$

$x_{ub} = \cos^{-1}\frac{5.365}{8.225} = 49.438°$

$cos x_{lb} = \frac{5.355}{8.235}$

$x_{lb} = \cos^{-1}\frac{5.355}{8.235} = 49.286°$

$x = 49°$ (to the nearest degree)

Finding a missing side

Step 1: Label your sides and angles

Step 2: Substitute known values into the formula

Step 3: Rearrange the formula to find the missing side

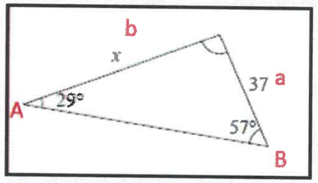
Find the length of side x.

$\frac{37}{sin29} = \frac{x}{sin57}$

$\frac{37 \times sin57}{sin29} = x$

$x = 64.004°$

$x = 64°$



Sine Rule

The Sine and Cosine Rules are used for finding missing sides and angles on non right angled triangles.

Finding a missing angle

$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$

$\frac{SinA}{a} = \frac{SinB}{b}$

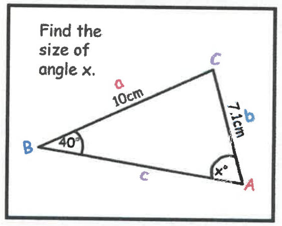
$\frac{SinA}{10} = \frac{Sin40}{7.1}$

$SinA = \frac{Sin40}{7.1} \times 10$

$A = \sin^{-1}\frac{Sin40}{7.1} \times 10$

$A = 64.9°$

Find the size of angle x.



The formula for the sine rule is

$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$

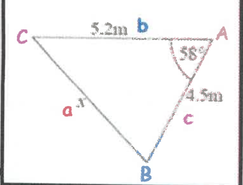
Cosine Rule: Finding a missing side

Can be used to find missing sides or angles. MUST have 2 sides and 1 angle

Formula for missing side

$a^2 = b^2 + c^2 - 2bcCosA$

Find the length of side x.



$x^2 = 5.2^2 + 4.5^2 - (2 \times 5.2 \times 4.5 \times Cos58)$

$x^2 = 5.2^2 + 4.5^2 - (24.8)$

$x^2 = 27.04 + 20.25 - (24.8)$

$x^2 = 47.29 - (24.8)$

$x^2 = 22.49$ Square root both sides

$x = 4.74m$

Cosine Rule: Finding a missing angle

Formula for missing angle

$$CosA = \frac{b^2 + c^2 - a^2}{2bc}$$

To find a missing angle… MUST have all 3 sides given

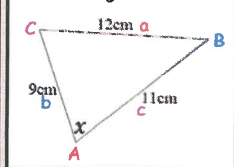
$Cosx = \frac{9^2 + 11^2 - 12^2}{2 \times 9 \times 11}$

$Cosx = \frac{58}{198}$

$x = \cos^{-1}\frac{58}{198}$

$x = 72.97°$

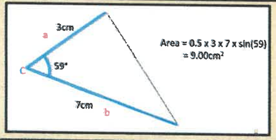
Find the size of angle x.



Area of a triangle

$$\text{Area} = \frac{1}{2} a\, b \sin(C)$$

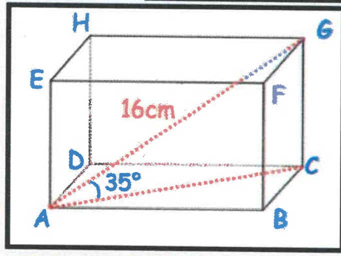
Where C is the angle wedged between two sides a and b.



Cosine Rule

Length AG = 16cm
Angle CAG is 35°

Work out the length of EG.



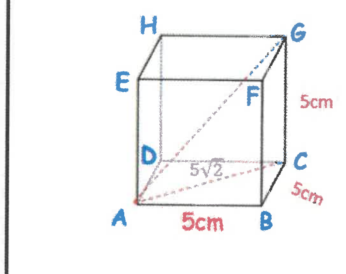
$\cos(CAG) = \frac{a}{h}$

$\cos(35) = \frac{EG}{16}$

$16 \times \cos(35) = EG$

EG = 13.1cm

1. Shown is a cube with side length 5cm.



Calculate angle CAG.

$AC^2 = \sqrt{5^2 + 5^2}$

$AC^2 = \sqrt{50}$

$AC = 5\sqrt{2}$

$\tan(CAG) = \frac{5}{5\sqrt{2}}$

$CAG = \tan^{-1}\frac{5}{5\sqrt{2}}$

$CAG = 35.3°$

Trigonometry in 3D

Unit 14: Further Statistics

Sampling

When you are investigating a hypothesis, the population is the whole group that you are interested in.

A population is everything or everybody that could possibly be involved in an investigation

Stratified Sampling

A population may divide into groups such as age range These groups are called strata.

In a stratified sample, the number of people taken from each group is proportional to the group size.

The grouped frequency table shows information about the weights, in kilograms, of 20 students, chosen at random from Year 11.

Weight (w kg)	Frequency
$50 \le w < 60$	7
$60 \le w < 70$	8
$70 \le w < 80$	3
$80 \le w < 90$	2

There are 300 students in Year 11.

Work out an estimate for the number of students in Year 11 whose weight is between 50 kg and 60 kg.

$\frac{7}{20} \times 300 = 105$

Capture Recapture

Capture-Recapture is a technique that can be used to estimate the total population.

$\frac{M}{N} = \frac{R}{T}$

M = total marked
N = total population
R = number "recaptured"
T = total captured on 2nd visit

Clive wants to estimate the number of bees in a beehive.
Clive catches 50 bees from the beehive.

He marks each bee with a dye.
He then lets the bees go.

The next day, Clive catches 40 bees from the beehive.
8 of these bees have been marked with the dye.

(i) Work out an estimate for the number of bees in the beehive.

M = 50
T = 40
R = 8
N = ?

$\frac{50}{n} = \frac{8}{40}$

$50 = \frac{8n}{40}$

$2000 = 8n$

$250 = n$

Cumulative Frequency

A Cumulative Frequency table shows a running total of the frequencies

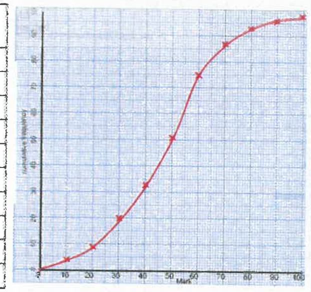
Mark	Frequency	Cumulative Frequency
1-10	3	3
11-20	6	3 + 6 = 9
21-30	11	9 + 11 = 20
31-40	13	20 + 13 = 33
41-50	18	33 + 18 = 51

A cumulative frequency table shows how many data values Are less than or equal to the upper class boundary of each data class

The upper class boundary is the highest possible value in each class.

Drawing a Cumulative Frequency Graph

Mark	Frequency	Cumulative Frequency
1-10	3	3
11-20	6	9
21-30	11	20
31-40	13	33
41-50	18	51
51-60	24	75
61-70	12	87
71-80	6	93
81-90	3	96
91-100	2	98



Steps:

1) Start from 0
2) Plot using end points
3) Join using a smooth curve

Box Plots + CF Diagrams

Averages

1 1 3 5 7 9 11 14 16 19 19 20 21

n = 13

Median $= \frac{13+1}{2} = \frac{14}{2} = 7th\ value = 11$

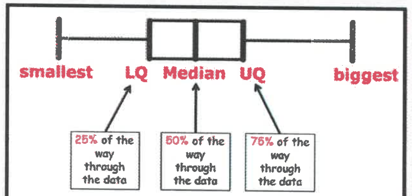
LQ $= \frac{13+1}{4} = \frac{14}{4} = 3.5th\ value = 4$

UQ $= 3.5 \times 3 = 10.5th\ value = 19$

Median formula $= \frac{n+1}{2}$

Lower Quartile formula $= \frac{n+1}{4}$

Upper Quartile formula $= LQ \times 3$

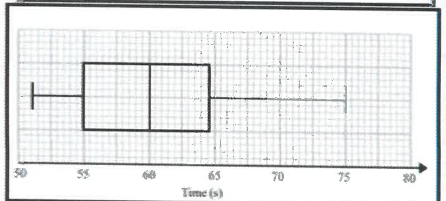


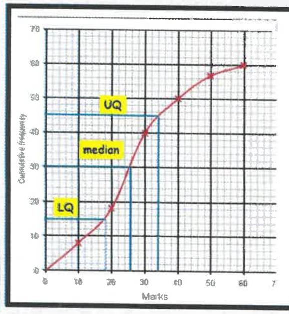
Drawing a box plot

The times, in seconds, of 15 students running a race are recorded below.

52 54 54 55 58 58 59 60 60 61 61 64 67 70 75

Draw a box plot for this information.





Median - Middle value found at total frequency ÷ 2

Lower quartile - Value which is a quarter of the way through the data found at total frequency ÷ 4

Upper quartile - Value which is three quarters of the way through the data Found at total frequency ÷ 4 x 3

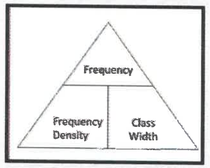
Interquartile range = Upper quartile - Lower quartile

Median = 26
Lower Quartile = 18
Upper Quartile = 34
IQR= 34 - 18 =16

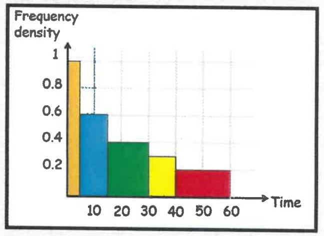
Histograms

Drawing a histogram

Time, t, in minutes	Frequency density	Frequency
$0 \le t < 5$	1	5
$5 \le t < 15$	0.6	6
$15 \le t < 30$	0.4	6
$30 \le t < 40$	0.3	3
$40 \le t < 60$	0.2	4



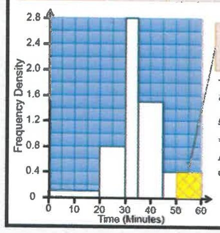
This is the formula triangle for a histogram



Interpreting a Histogram

The histogram shows the times a sample of students spent on the internet one evening. (a) Copy and complete the frequency table,
(b) Estimate how many students spent longer than 50 minutes on the internet.

Time, t, minutes	$0 \le t < 20$	$20 \le t < 30$	$30 \le t < 35$	$35 \le t < 45$	$45 \le t < 60$
Frequency	0.1 x 20 = 2	0.8 x 10 = 8	2.8 x 5 = 14	1.5 x 10 = 15	0.4 x 15 = 6



50 Minutes or more

The area of the histogram represents area.

Area $50 \le t < 60$

= 0.4 x 10 = 4

4 students spent longer than 50 minutes on the internet.

Unit 18: Vectors

Vector Basics

What is a vector?

A vector describes *direction* and *length*

The magnitude of a vector is its size

$\begin{pmatrix} x \\ y \end{pmatrix}$

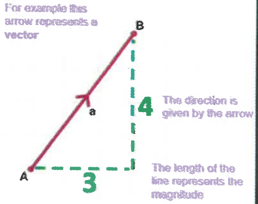
X = number of moves to the **right** or **left**

Y= number of moves **up** or **down**

This vector can be written in 3 ways

a $\bar{a}$ $\overrightarrow{AB}$

For example this arrow represents a vector

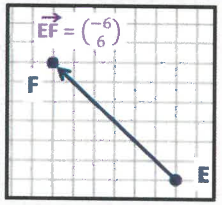


The direction is given by the arrow

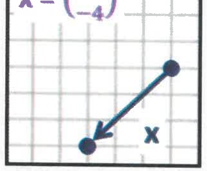
The length of the line represents the magnitude

$a = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\bar{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\overrightarrow{EF} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$



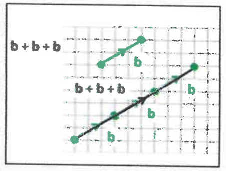
$x = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$



Vector Arithmetic

What's another way of saying b + b + b?

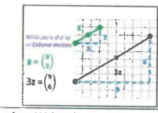
3b — Scalar — Vector



Multiplying a Vector by a Scalar

$z = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$3z = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$



To multiply a Vector by a Scalar. Write the Vector as a Column Vector then multiply each entry in the Column Vector by the Scalar

$3z = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{matrix} \times 3 \\ \times 3 \end{matrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

We can multiply a vector by a scalar

A scalar is a quantity that has size but no direction

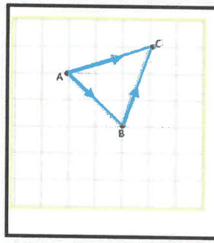
Vectors that have been multiplied by a scalar are parallel

If we add two or more vectors together we get a resultant vector

A resultant vector is the vector sum of two or more vectors

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$



$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\overrightarrow{BC} - \overrightarrow{AB} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

Midpoints and Ratio

Midpoints of Vectors

3. *P* is the point (1,5), *Q* is the point (9,3)

a) Write down the vector $\overrightarrow{PQ}$
Write your answer as a column vector $\begin{pmatrix} 8 \\ -2 \end{pmatrix}$

M is the midpoint of PQ

$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

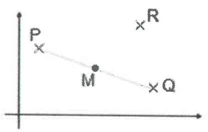
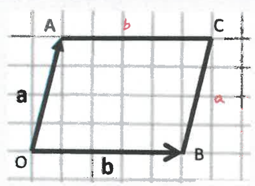


Diagram **NOT** accurately drawn

Vectors in quadrilaterals

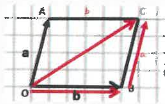


OACB is a parallelogram

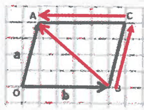
$\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$

Find i) $\overrightarrow{OC}$ ii) $\overrightarrow{BA}$ iii) $\overrightarrow{CA}$

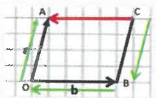
In terms of a and b



$\overrightarrow{OC} = b + a$



$\overrightarrow{BA} = a - b$



$\overrightarrow{CA} = -b$

$\overrightarrow{CA} = -a - b + a$

$\overrightarrow{CA} = -b$

Vectors with ratio

$\overrightarrow{AL} : \overrightarrow{LB} = 2 : 1$

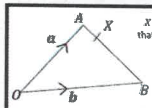
What is:

$\overrightarrow{AB} = a$

$\overrightarrow{AL} = \frac{2}{3}a$

$\overrightarrow{LB} = \frac{1}{3}a$

There are 3 parts to the ratio So we are dealing with thirds



X is a point such that $AX : XB = 1:4$

a. $\overrightarrow{AB} = -a + b$

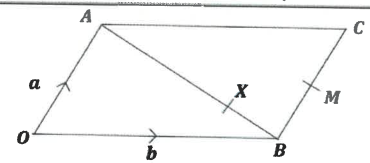
b. $\overrightarrow{AX} = -\frac{1}{5}a + \frac{1}{5}b$

c. $\overrightarrow{OX} = \frac{4}{5}a + \frac{1}{5}b$

d. $\overrightarrow{BX} = \frac{4}{5}a - \frac{4}{5}b$

Vector Proof

How to show two vectors are parallel



X is a point on *AB* such that $AX : XB = 3:1$. *M* is the midpoint of *BC*.
Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.

$\overrightarrow{OC} = a + b$

$\overrightarrow{XM} = \frac{1}{4}(-a + b) + \frac{1}{2}a = \frac{1}{4}a + \frac{1}{4}b$

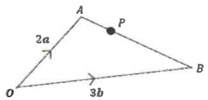
$= \frac{1}{4}(a + b)$

$\overrightarrow{XM}$ **is a multiple of** $\overrightarrow{OC}$ ∴ parallel.

For any proof question always find the vectors involved first, in this case $\overrightarrow{XM}$ and $\overrightarrow{OC}$.

The key is to factor out a scalar such that we see the same vector.

The magic words here are "*is a multiple of*".



a) Find $\overrightarrow{AB}$ in terms of *a* and *b*.

$-2a + 3b$

b) *P* is the point on *AB* such that *AP* : *PB* = 2 : 3.
Show that $\overrightarrow{OP}$ is parallel to the vector $a + b$.

M1 for $2a \pm \frac{2}{5}(3b - 2a)$ OR $3b \pm \frac{3}{5}(2a - 3b)$

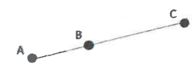
A1 for $\frac{6}{5}a + \frac{6}{5}b$ oe

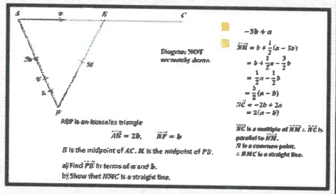
A1 for $\frac{6}{5}(a + b)$ is parallel to a + b oe

Collinear Points

Points A, B and C form a straight line if:
$\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel (and B is a common point).

Alternatively, we could show $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel. This tends to be easier.





Unit 17: Further Algebra

Algebraic Fractions

When we multiply fractions we multiply and the numerators together and the denominators together

If the fraction shares common factors then we can cross cancel by dividing the factors out before we multiply

$\frac{7}{11} \times \frac{55}{7} = \frac{1}{1} \times \frac{5}{1} = 5$

$\frac{a^2}{2} \times \frac{4}{a} = \frac{a}{1} \times \frac{2}{1} = \frac{2a}{1} = 2a$

$\frac{3(c+2)}{8c} \times \frac{2c(c-1)}{c+2} = \frac{3}{4} \times \frac{c-1}{1} = \frac{3(c-1)}{4}$

When we divide fractions we find the reciprocal of the second fraction and then multiply the fractions together.

$\frac{6x}{2y} \div \frac{4y}{5} = \frac{6x}{2y} \times \frac{5}{4y} = \frac{15}{4y^2}$

When we add or subtract with fractions we need to have the same denominator in each fraction.

To do this we need to find the lowest common denominator

$\frac{x}{3} + \frac{2x+1}{2} = \frac{2x + 3(2x+1)}{3 \times 2} = \frac{2x + 6x + 3}{6} = \frac{8x+3}{6}$

LCM = 6

Step 1: Find the lowest common denominator

Step 2: Cross multiply

Step 3: Simplify

Solve $\frac{x+5}{7} = 5$

$\times 7 \quad \times 7$

$x + 5 = 35$

$-5 \quad -5$

$x = 30$

Solve the equation $\frac{x+4}{2} = \frac{x+10}{3}$ ($\times 3$, $\times 2$)

$3(x+4) = 2(x+10)$

$3x + 12 = 2x + 20$ (-12, -12)

$3x = 2x + 8$ ($-x$, $-x$)

$x = 8$

Solve

$\frac{2x+1}{3} + \frac{x-5}{2} = 4$ LCD = 6

$\frac{2(2x+1) + 3(x-5)}{3 \times 2} = 4$

$\frac{4x + 4 + 3x - 15}{6} = 4$

$\frac{7x - 11}{6} = 4$ ($\times 6$, $\times 6$)

$7x - 11 = 24$ ($+11$, $+11$)

$7x = 35$ ($\div 7$, $\div 7$)

$x = 5$

Step 1: Find the lowest common denominator

Step 2: Cross Multiply

Step 3: Simplify the numerator

Step 4: Solve

Changing the subject

1. Circle the subject that we want to isolate
2. Identify what needs to be eliminated
3. Inverse the operation, and apply to both sides

When we have two terms on one side:

1) Factorise out the subject
2) Circle the subject that we want to isolate
3) Identify what needs to be eliminated
4) Inverse the operation, and apply both sides

Make x the subject of:

$ax + b = cx + d$

$ax - cx = d - b$

$x(a - c) = d - b$

$x = \frac{d-b}{a-c}$

Make c the subject

$cm + ca = z$

$c(m + a) = z$

$\div m + a \quad \div m + a$

$c = \frac{z}{m+a}$

Make x the subject of:

$q = \frac{2x+1}{2x-1}$

$q(2x - 1) = 2x + 1$

$2xq - q = 2x + 1$

$2xq - 2x = 1 + q$

$x(2q - 2) = 1 + q$

$x = \frac{1+q}{2q-2}$

Simplifying Surds

Simplify:

$\sqrt{60}$

$\sqrt{4 \times 15}$

$\sqrt{4}\sqrt{15}$

$2\sqrt{15}$

1) Find two factors of the number (one of them needs to be a square number)

2) Chop the surd in half

3) Simplify

$\sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2}$

Simplify $\sqrt{2} + \sqrt{8}$

When the numbers under the square root are different we have to simplify first

$= \sqrt{2} + \sqrt{4}\sqrt{2}$

$= \sqrt{2} + 2\sqrt{2}$

$= 3\sqrt{2}$

Surds

Multiplying Surds

$\sqrt{5} \times \sqrt{6} = \sqrt{30}$

$(2 + \sqrt{3})(2 + \sqrt{3})$

x	2	$\sqrt{3}$
2	4	$2\sqrt{3}$
$\sqrt{3}$	$2\sqrt{3}$	$\sqrt{9}$

$= 4 + 2\sqrt{3} + 2\sqrt{3} + \sqrt{9}$

$= 4 + 4\sqrt{3} + 3$

$= 7 + 4\sqrt{3}$

Rationalising the denominator

Rationalise the denominator for $\frac{3}{\sqrt{5}}$

$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

Rationalise the denominator for $\frac{2}{\sqrt{3}+1}$

$= \frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{2\sqrt{3}-2}{3 - \sqrt{3} + \sqrt{3} - 1}$

$= \frac{2\sqrt{3}-2}{2}$

$= \sqrt{3} - 1$

Algebraic Proof

SHOW...

"SHOW THAT THE SUM OF TWO CONSECUTIVE NUMBERS IS ALWAYS AN ODD NUMBER"

1 + 2 = 3 ✓

2 + 3 = 5 ✓

5 + 6 = 11 ✓

101 + 102 = 203 ✓

We have SHOWN that this works. We have not proved it works.

There could be, at some point, two consecutive numbers in fact give us an EVEN number

PROVE...

Algebra is often used to establish the truth (to prove something)

"PROVE THAT THE SUM OF TWO CONSECUTIVE NUMBERS IS ALWAYS AN ODD NUMBER"

We can give any number, any letter. In this case, lets pick the letter "n"

Therefore the number directly after n is "n+1"

"SUM OF TWO CONSECUTIVE NUMBERS."

n + n + 1 = 2n + 1

"2n" is the nth term for the multiples of 2.

2n + 1 is one more than the multiples of 2, the odd numbers.

This PROVES that the statement is always true for any value of n

Identity:

An identity in maths is represented by the ≡ symbol

An identity is an equation which is always true no matter which values are chosen

$(x + 1)^2 \equiv x^2 + 2x + 1$

Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4 for all positive integer values of n"

"Before you begin, highlight the key words and information from the question

Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4 for all positive integer values of n"

STEP 1: EXPAND

$(3n + 1)^2 - (3n - 1)^2$

$= (3n + 1)(3n + 1) - (3n - 1)(3n - 1)$

$= (9n^2 + 6n + 1) - (9n^2 - 6n + 1)$

STEP 2: SIMPLIFY

$= (9n^2 + 6n + 1) - (9n^2 - 6n + 1)$

$= 12n$

STEP 3: FACTORISE

$12n = 4 \times 3n$

STEP 3: JUSTIFY

$4 \times 3n$ → always divisible by 4 and hence a multiple of 4

Prove that

$(n + 1)^2 - (n - 1)^2 + 1$

is always odd for all positive integer values of n

Expand

$= (n + 1)(n + 1) - (n - 1)(n - 1) + 1$

$= (n^2 + 2n + 1) - (n^2 - 2n + 1) + 1$

Simplify

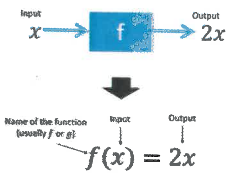
$= 4n + 1$

Justify

$4 \times n$ is always even

Any even number add 1 is odd

Functions



If $p = 3x$ and $x = \frac{y}{2}$

Write p in terms of y

$p = 3x$

$p = 3\left(\frac{y}{2}\right)$

$p = \frac{3y}{2}$

You are given that $f(x) = 3x + 1$

Find $f(1)$

This means we need to substitute x for 1

Find $f(1) = 3(1) + 1 = 3 + 1 = 4$

$f(x) = x^2 + 2$

If $f(a) = 38$, what is a?

$x^2 + 2 = 38$

$x^2 = 36$

$x = \pm 6$

If $f(x) = x + 1$ what is:

$f(x + 1) = (x + 1) + 1 = x + 2$

$f(x - 1) = (x - 1) + 1 = x$

$f(x^2) = x^2 + 1$

$f(x)^2 = (x + 1)^2 = x^2 + 2x + 1$

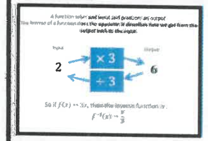
$f(2x) = 2x + 1$

$2f(x) = 2(x + 1) = 2x + 2$

Anything inside the bracket applies just to the x

Anything outside the bracket applies just to the whole function

Inverse Functions



STEP 1: Write the output $f(x)$ as y

STEP 2: Get the input in terms of the output (make x the subject).

STEP 3: Swap y back for x and x back for $f^{-1}(x)$.

$f(x) = 2x + 1$

find $f^{-1}(x)$

$y = 2x + 1$

$y - 1 = 2x$

$x = \frac{y-1}{2} \quad f^{-1}(x) = \frac{x-1}{2}$

Composite Functions

Given $f(x) = 2x$

$g(x) = x + 1$

find $fg(2)$

$g(2) = 2 + 1 = 3$

$f(3) = 2 \times 3 = 6$

Given $f(x) = 2x$

$g(x) = x + 1$

find $fg(1)$

$g(1) = 1 + 1 = 2$

$f(2) = 2 \times 2 = 4$

$f(x) = x + 1$

$g(x) = 2x$

Find $gf(x)$

$f(x) = x + 1$

$g(x + 1) = 2(x + 1) = 2x + 2$

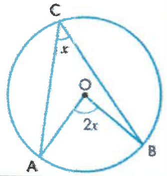
$gf(x) = 2x + 2$

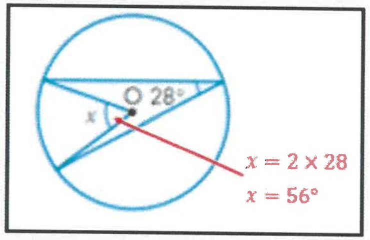
Unit 16: Circle Theorems

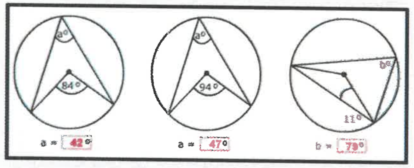
Circle Theorem 1:

"The angle at the centre of a circle is twice the angle at the circumference."

Angle AOB = 2 x ACB

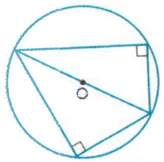


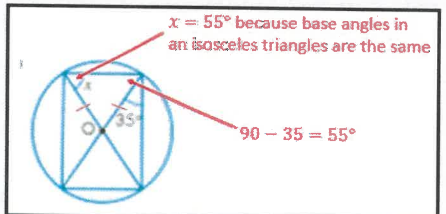


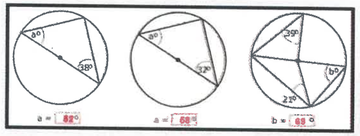


Circle Theorem 2:

"Every angle at the circumference of a semicircle that is subtended by the diameter of the semicircle is a right-angle."



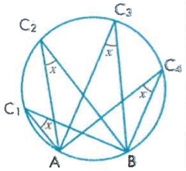


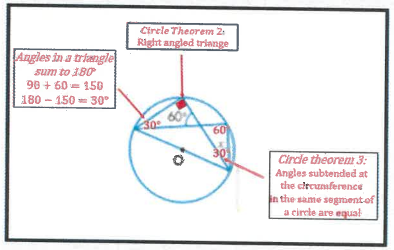


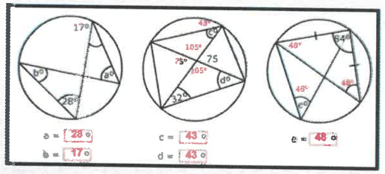
Circle Theorem 3:

"Angles subtended at the circumference in the same segment of a circle are equal."

Points C_1, C_2, C_3 and C_4 on the circumference are subtended by the same arc, AB.



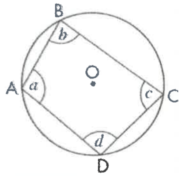


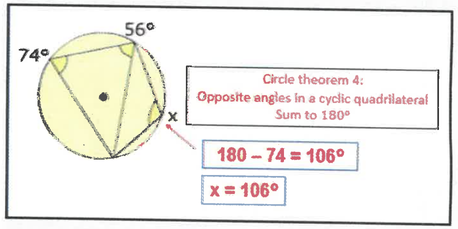


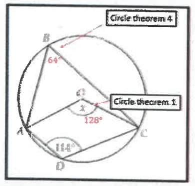
Circle Theorem 4:

"The sum of the opposite angles in a cyclic quadrilateral is 180°."

a + c = 180° and b + d = 180°

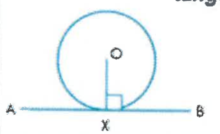


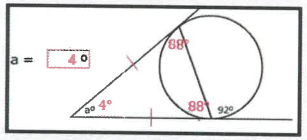




Circle Theorem 5:

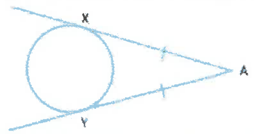
"When a radius meets a tangent, it always makes a 90° angle."





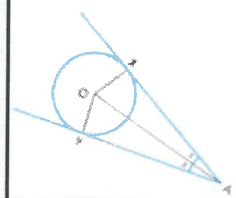
Circle Theorem 6:

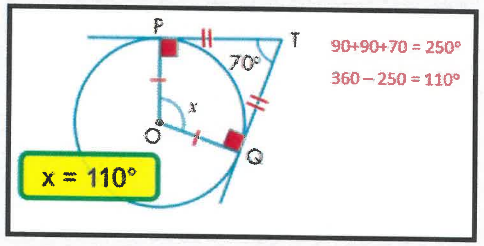
"Tangents to a circle from an external point to the points of contact are equal in length."



Circle Theorem 7:

"The line joining an external point to the centre of the circle bisects the angle between the tangents."

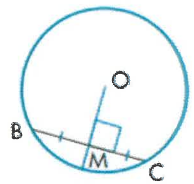


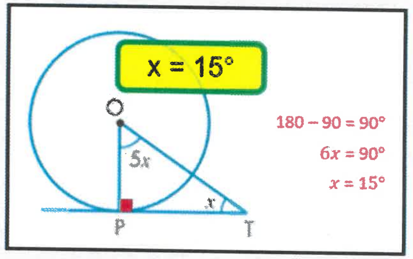


Circle Theorem 8:

"A radius bisects a chord at 90°."

If O is the centre of the circle, angle BMO = 90° and BM = CM.

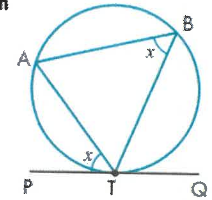


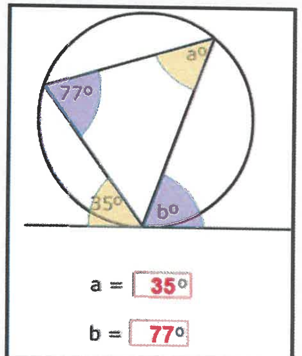


Circle Theorem 9:

The Alternate Segment Theorem

"The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment."





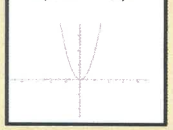
Unit 15: Equations and Graphs

Plotting Quadratic Graphs

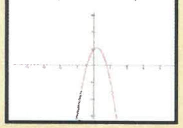
Quadratic Graphs

Quadratic graphs are curved and symmetrical

Positive Quadratic Graphs have a U shape

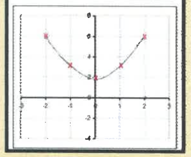


Negative Quadratic Graphs have a ∩ shape

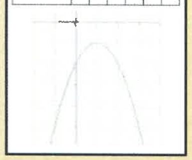


When you square a negative number the answer is always positive

x	-2	-1	0	1	2
$y = x^2 + 2$	6	3	2	3	6



x	-2	-1	0	1	2	3
$y = -x^2 + 2x - 2$	-10	-5	-2	-1	-2	-5



Finding the roots of a Quadratic Graph

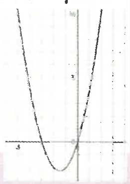
To find the roots of a graph we factorise

Find the roots for this equation

$x^2 + 3x = 0$

$x(x + 3) = 0$

$x = 0 \text{ or } x = -3$



Steps:
1) Set the equation equal to 0
2) Factorise
3) Solve for x

Finding the turning points of a Quadratic Graph

Find the turning point for the equation

$y = x^2 + 2x - 3$

Will the graph have a minimum or a maximum turning point?

Minimum

What are the coordinates of the turning point?

$(x + 1)^2 - 4$

To find a maximum or minimum point you complete the square

Simultaneous Equations

Solving simultaneous equations by using the graph

The diagram shows the graphs of

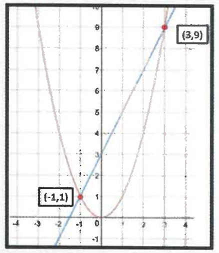
$y = x^2$

$y = 2x + 3$

Use the diagram to solve this pair of simultaneous equations:

$y = x^2$

$y = 2x + 3$



The points where the graphs intersect are the solutions of the simultaneous equations.

$x = -1, y = 1 \text{ or } x = 3, y = 9$

Solving simultaneous equations by plotting the graphs

Sketch the graph of $y = x^2 - 3$

x	-3	-2	-1	0	1	2	3
$y = x^2 - 3$	6	1	-2	-3	-2	1	6

By drawing a suitable line on your graph, solve this pair of simultaneous equations:

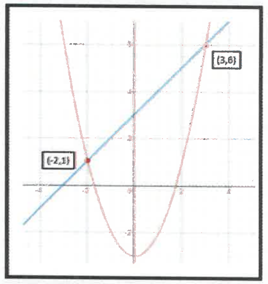
$y = x^2 - 3$

$y = x + 3$

x	-3	-2	-1	0	1	2	3
$y = x + 3$	0	1	2	3	4	5	6

The points where the graphs intersect are the solutions of the simultaneous equations.

Use a table of values to help you plot the graphs more accurately

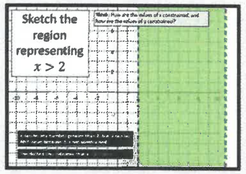


$x = -2, y = 1 \text{ or } x = 3, y = 6$

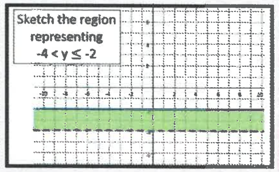
Graphical Inequality regions

Representing graphical inequalities

Sketch the region representing $x > 2$

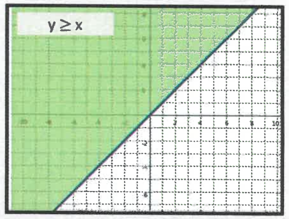


Sketch the region representing $-4 < y \leq -2$



If the line is a boundary for values that are included, the line must be drawn with a solid line

If the line is a boundary for values that are not included, the line must be drawn with a dashed line



Solving graphical inequalities

On the grid, shade the region whose coordinates satisfy the inequalities:

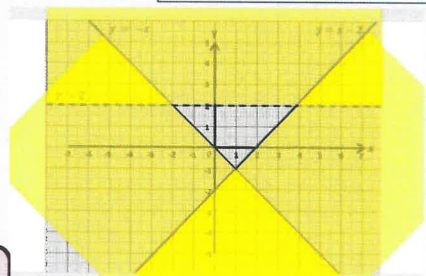
$y \geq -x$

$y < 2$

$y \geq x - 2$

To solve inequalities graphically

1. Draw the lines for the inequalities, treating them as equations (remember solid or dashed lines!)
2. Choose a point on either side of the line to test if the inequality is true or not
3. Shade the region that satisfies each inequality
4. The solution will be the unshaded region



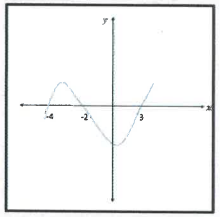
Cubic Graphs

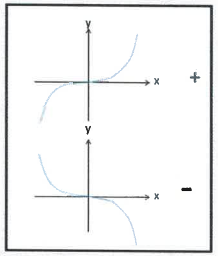
Sketching cubic graphs

Sketch the cubic graph

$(x + 2)(x - 3)(x + 4)$

Set each bracket to 0 this will give you the roots





Positive cubic graph +

Negative cubic graph −

Finding roots of cubic graphs by iteration

a) Show that the equation $x^3 - x - 19 = 0$ can be arranged to give $x = \sqrt[3]{x + 19}$

$x^3 = x + 19$

$x = \sqrt[3]{x + 19}$

b) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \sqrt[3]{x_n + 19}$ four times to find an approximate solution to $x^3 - x - 19 = 0$
Write all the digits on your calculator display.

$x_0 = 0$

$x_1 = 2.6680401 \ldots$

$x_2 = 2.7878899 \ldots$

$x_3 = 2.7930050 \ldots$

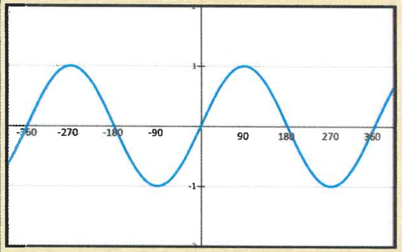
$x_4 = 2.7932236 \ldots$

$x \approx 2.79$

Unit 13a: Trigonometric Graphs

The graphs of sine, cos and Tan

Sine Graph



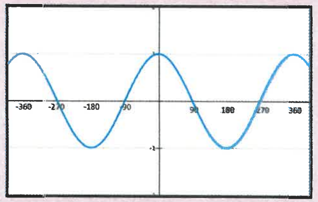
The Sine Graph repeats every 360°

It's largest value is 1

It's smallest value is -1

Each graph is shown between the range of –360 degrees and + 360 degrees. The range can be changed/restricted so make sure you read the range carefully in a question.

Cos Graph



The Cosine Graph repeats every 360°

It's largest value is 1

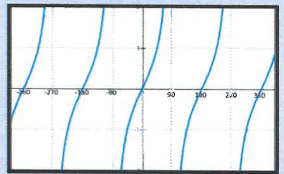
It's smallest value is -1

Tan Graph

The Tan Graph repeats every 180°

There are asymptotes (the dashed lines) that the tan graph never touches.

It doesn't have a maximum or minimum point, it goes between negative infinity and positive infinity.



Solving Equations with Trig Graphs

Steps

1) Sketch the trig graph within the specified range.
2) Draw a straight horizontal line at your given value notice how many times it crosses the graph (this is how many answers there are)
3) Substitute the value given into the **inverse trig function** into your calculator.
4) Round to a suitable degree of accuracy
5) This gives you **one** answer
6) Use your answer to find other solutions of the equation.

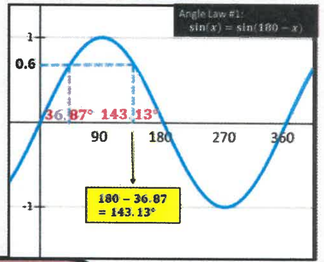
Solve sin(x)=0.6 in the range 0≤x<360

1) Draw a straight horizontal line at 0.6 (you can see we will have 2 solutions)

2) Substitute 0.6 into the **inverse trig function** into your calculator

3) Round to a suitable degree of accuracy: 36.87°

4) Subtract 36.87° from 180° to find the second solution.



$x = \sin^{-1}(0.6) = 36.87°, 143.13°$

Solve $3\cos(x) = 2$ in the range $0 \le x < 360$

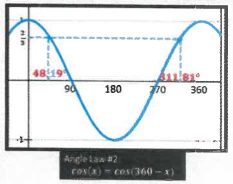
1) Rearrange first to eliminate the 3

2) Draw a horizontal line at $\frac{2}{3}$

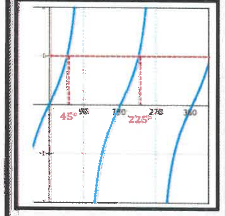
3) Substitute $\frac{2}{3}$ into the inverse cos function

4) Round the answer to a suitable degree of accuracy

5) Find all solutions



$x = \cos^{-1}\left(\frac{2}{3}\right) = 48.19°, 311.81°$



Solve $\tan(x) = 1$ in the range $0 \le x < 360$

$x = \tan^{-1}(1) = 45°$

$45° + 180° = 225°$

1) Draw a horizontal line at 1

2) Substitute 1 into the inverse tan function

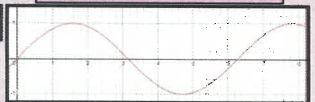
3) Find all solutions

Transformations of Graphs

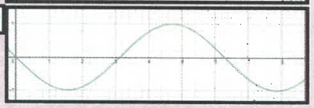
When we have a negative change inside the bracket

$f(-x)$ *is a reflection of the graph in the y – axis*

$y = \sin x$



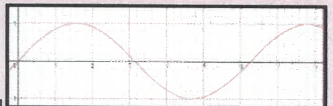
$y = \sin(-x)$



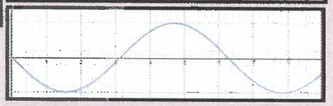
When we have a negative change outside the bracket

$-f(x)$ *is a reflection of the graph in the x – axis*

$y = \sin x$



$y = -\sin x$



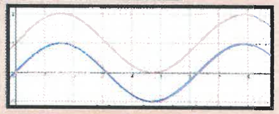
When we add or subtract outside the bracket it translates the graph in y

$f(x) + 1$ *translates the graph by* $\begin{pmatrix}0\\1\end{pmatrix}$

$f(x) - 1$ *translates the graph by* $\begin{pmatrix}0\\-1\end{pmatrix}$

$y = \sin x$

$y = \sin x + 1$



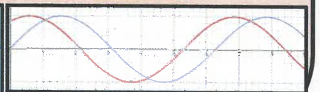
When we add or subtract inside the bracket it translates the graph in x (in the opposite direction)

$f(x+1)$ *translates the graph by* $\begin{pmatrix}-1\\0\end{pmatrix}$

$f(x-1)$ *translates the graph by* $\begin{pmatrix}1\\0\end{pmatrix}$

$y = \sin x$

$y = \sin(x+1)$

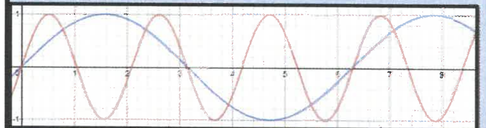


When we have a change inside the bracket

$f(ax)$ *squashes on the x axis by a factor of a*

$y = \sin x$

$y = \sin 3x$



f(3x) → Squash on x-axis by factor of 3

When we have a change outside the bracket

$af(x)$ *stretched on the y – axis by a factor of a*

2f(x) → Stretch on y-axis by factor of 2

$y = \sin x$

$y = 2\sin x$