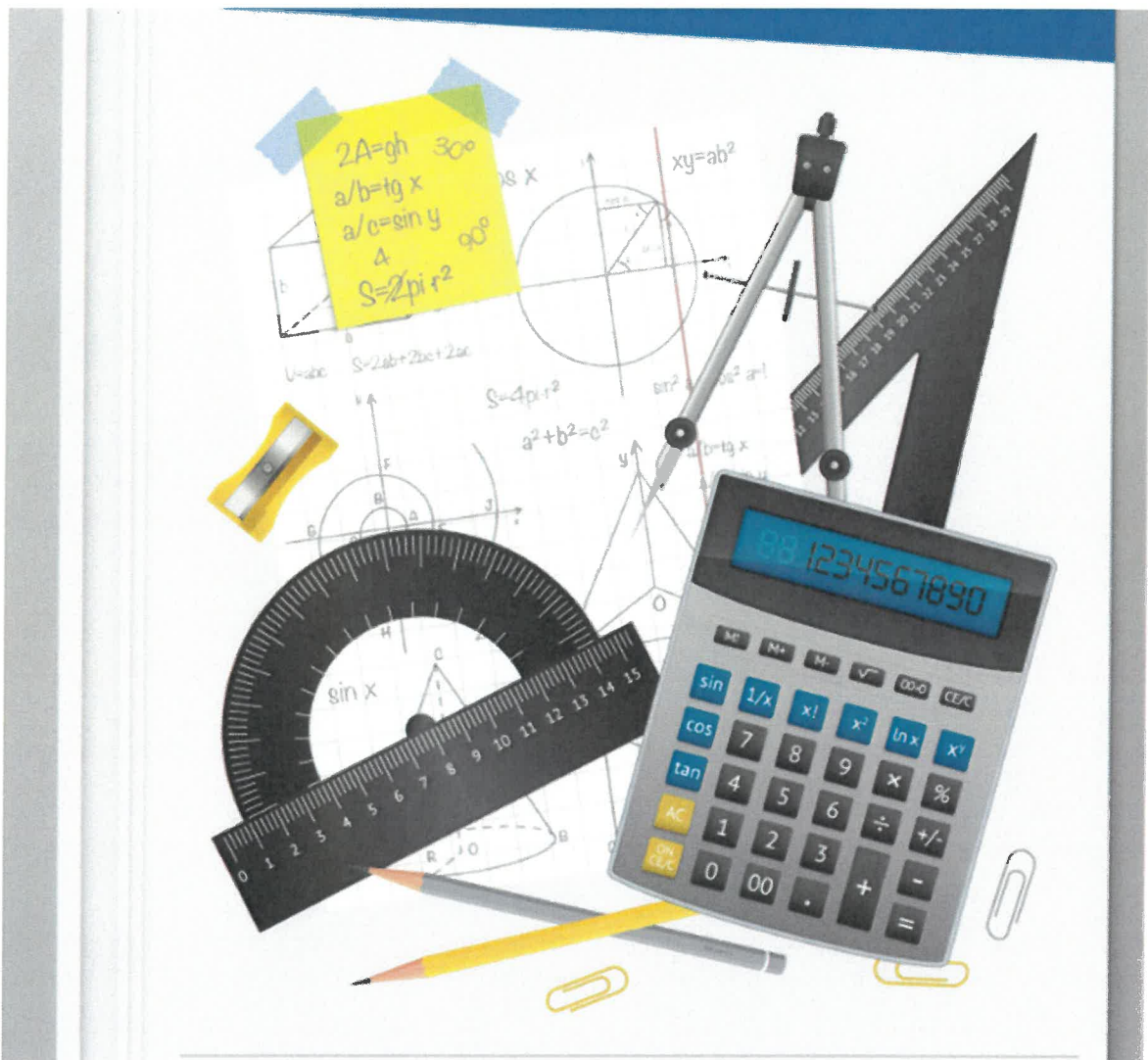


# Madani Girls Maths Knowledge Organiser



# How do we revise with our Knowledge Organisers?

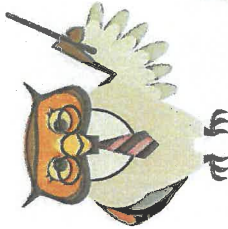
## Record It

Record yourself on your phone or tablet reading out the information. These can be listened to as many times as you want!



## Teach it!

Teach someone your key facts and the get them to test you, or even test them!



## Flash Cards

Write the key word or date on one side and the explanation on the other. Test your memory by asking someone to quiz you on either side.



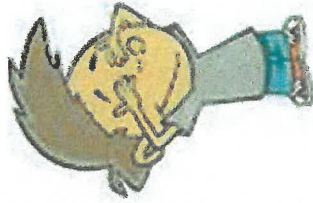
## Back to front

Write down the answers and then write out what the questions the teacher may ask to get those answers.



## Hide and Seek

Read through your knowledge organiser, put it down and try and write out as much as you can remember. Then keep adding to it until its full!



## Sketch it

Draw pictures to represent each of the facts or dates. It could be a simple drawing or something that reminds you of the answer.



## Post its

Using a pack of post-it notes, write out as many of the keywords or dates as you can remember in only 1 minute!

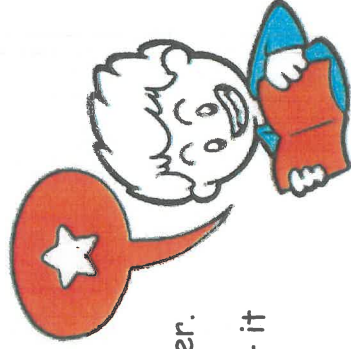


## Practice!

Some find they remember by simply writing the facts over and over again.

## Read Aloud

Simply speak the facts and dates out loud as you're reading the Knowledge Organiser. Even try to act out some of the facts - it really helps you remember!





# USING NUMBER...

## Non-calculator methods

@whisto\_maths

### What do I need to be able to do?

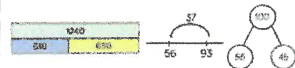
By the end of this unit you should be able to:

- Use mental/written methods for the four number operations
- Use four operations for fractions
- Write exact answers
- Round to decimal places and significant figures
- Estimate solutions
- Understand limits of accuracy
- Understand financial maths

### Keywords

- Truncate:** to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)
- Round:** making a number simpler, but keeping its place value close to what it originally was
- Credit:** money that goes into a bank account
- Debit:** money that leaves a bank account
- Profit:** the amount of money after income - costs
- Tax:** money that the government collects based on income, sales and other activities.
- Balance:** The amount of money in a bank account
- Overestimate:** Rounding up - gives a solution higher than the actual value
- Underestimate:** Rounding down - gives a solution lower than the actual value

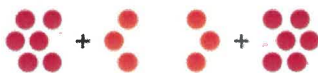
### Addition/ Subtraction



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

|   | H | T | O |
|---|---|---|---|
|   | 1 | 8 | 7 |
| + | 5 | 4 | 2 |
|   |   |   |   |

|   | H | T | O |
|---|---|---|---|
|   | 4 | 2 | 7 |
| - | 2 | 4 | 9 |
|   |   |   |   |

Remember the place value of each column  
You may need to move 10 ones to the ones column to be able to subtract

Decimals have the same methods remember to align the place value

### Division methods

Short division

$$\begin{array}{r} 512 \\ 7 \overline{) 3584} \\ \underline{35} \phantom{00} \\ 0 \phantom{00} \\ \underline{0} \phantom{00} \\ 0 \phantom{00} \\ \underline{0} \phantom{00} \\ 0 \phantom{00} \end{array}$$

Complex division

$$\div 24 = \div 6 \div 4$$

Break up the divisor using factors

$$3584 \div 7 = 512$$

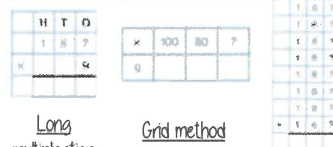
Division with decimals

The placeholder in division methods is essential - the decimal lines up on the dividend and the quotient

$$24 \div 0.02 \longrightarrow 24 \div 0.2 \longrightarrow 240 \div 2$$

All give the same solution as represent the same proportion  
Multiply the values in proportion until the divisor becomes an integer

### Multiplication methods



Long multiplication (column)

Grid method

Repeated addition

Less effective method especially for bigger multiplication

Multiplication with decimals

Perform multiplications as integers  
e.g.  $0.2 \times 0.3 \longrightarrow 2 \times 3$

Make adjustments to your answer to match the question:  
 $0.2 \times 10 = 2$   
 $0.3 \times 10 = 3$

Therefore  $6 \div 100 = 0.06$

### Four operations with fractions

Addition and Subtraction

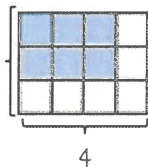
$$\frac{4}{5} - \frac{2}{3}$$



$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Multiplication

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$



Division

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

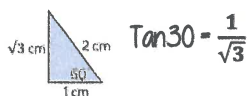
$$= \frac{8}{15}$$

### Exact Values

Leave in terms of  $\pi$

$$\frac{120}{360} \times 36\pi = \frac{1}{3} \times 36\pi = 12\pi$$

Leave as a surd



$$\tan 30 = \frac{1}{\sqrt{3}}$$

### Rounding

2.46192 (to 1dp) - is this closer to 2.46 or 2.47

$$2.46 \mid 192$$

This shows the number is closer to 2.46

Significant Figures

- 370 to 1 significant figure is 400
- 37 to 1 significant figure is 40
- 3.7 to 1 significant figure is 4
- 0.37 to 1 significant figure is 0.4
- 0.00000037 to 1 significant figure is 0.0000004

SF: Round to the first nonzero number

### Estimation

Round to 1 significant figure to estimate  
 $21.4 \times 3.1 \approx 20 \times 3 \approx 60$

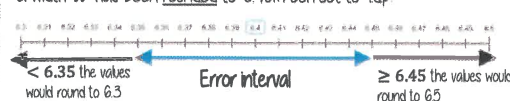
The equal sign changes to show it is an estimation

This is an underestimate because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths - it helps you identify calculation errors.

### Limits of accuracy

A width  $w$  has been rounded to 6.4cm correct to 1dp

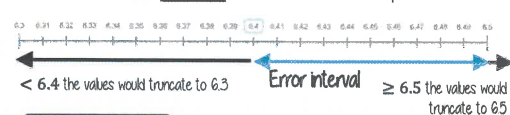


The error interval

$$6.35 \leq w < 6.45$$

Any value within these limits would round to 6.4 to 1dp

A width  $w$  has been truncated to 6.4cm correct to 1dp



$$6.4 \leq w < 6.5$$

Any value within these limits would truncate to 6.4 to 1dp

# USING NUMBER...

## Types of number & sequences

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Understand factors and multiples
- Express numbers as a product of primes
- Find the HCF and LCM
- Describe and continue sequences
- Explore sequences
- Find the  $n$ th term of a linear sequence

### Keywords

**Factor:** numbers we multiply together to make another number

**Multiple:** the result of multiplying a number by an integer.

**HCF:** highest common factor. The biggest factor that numbers share.

**LCM:** lowest common multiple. The first multiple numbers share.

**Arithmetic:** a sequence where the difference between the terms is constant

**Geometric:** a sequence where each term is found by multiplying the previous one by a fixed nonzero number

**Sequence:** items or numbers put in a pre-decided order

### Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

Non example of a multiple

4.5 is not a multiple of 3 because it is  $3 \times 1.5$

Not an integer

$x$  can take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

### Factors

Arrays can help represent factors

$5 \times 2$  or  $2 \times 5$

Factors of 10  
1, 2, 5, 10

$10 \times 1$  or  $1 \times 10$

Factors and expressions

$x \times x \times x \times x \times x$

The number itself is always a factor

Factors of  $6x$

$6, x, 1, 6x, 2x, 3, 3x, 2$

$6x \times 1$  OR  $6 \times x$

$x \times x$   
 $x \times x$

$2x \times 3$

$x \times x \times x$   
 $x \times x \times x$

$3x \times 2$

### Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number  
The only even prime number

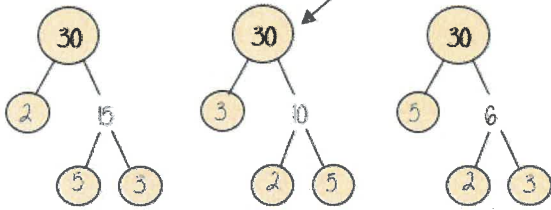
2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

### Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

eg 60  $30 \times 2$   $2 \times 3 \times 5 \times 2$   
150  $30 \times 5$   $2 \times 3 \times 5 \times 5$

### Finding the HCF and LCM

HCF - Highest common factor

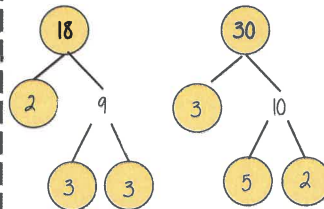
HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share

HCF = 6



LCM - Lowest common multiple

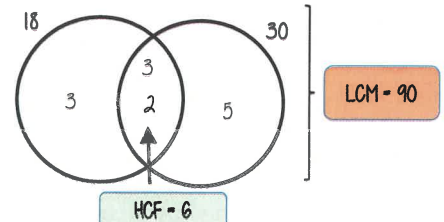
LCM of 18 and 30

18: 18, 36, 54, 72, 90

30: 30, 60, 90

The first time their multiples match

LCM = 90



### Arithmetic/ Geometric sequences

**Arithmetic Sequences** change by a common difference. This is found by addition or subtraction between terms

**Geometric Sequences** change by a common ratio. This is found by multiplication/ division between terms.

**Term to term rule** - how you get from one term (number in the sequence) to the next term.

**Position to term rule** - take the rule and substitute in a position to find a term. Eg. Multiply the position number by 3 and then add 2

### Other sequences

Fibonacci Sequence

1, 1, 2, 3, 5, 8...

Each term is the sum of the previous two terms

Triangular Numbers - look at the formation

1, 3, 6, 10, 15...

Square Numbers - look at the formation

1, 4, 9, 16...

Sequences are the repetition of a pattern

### Finding the nth term

This is the 4 times table  $\rightarrow 4, 8, 12, 16, 20 \dots$

$4n$

This has the same constant difference - but is 3 more than the original sequence

7, 11, 15, 19, 22

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence



# USING NUMBER...

## Indices & Roots

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

### Keywords

**Standard (index) Form:** A system of writing very big or very small numbers

**Commutative:** an operation is commutative if changing the order does not change the result

**Base:** The number that gets multiplied by a power

**Power:** The exponent – or the number that tells you how many times to use the number in multiplication

**Exponent:** The power – or the number that tells you how many times to use the number in multiplication

**Indices:** The power or the exponent

**Negative:** A value below zero.

**Coefficient:** The number used to multiply a variable

### Square and cube numbers

#### Square numbers

1, 4, 9, 16...

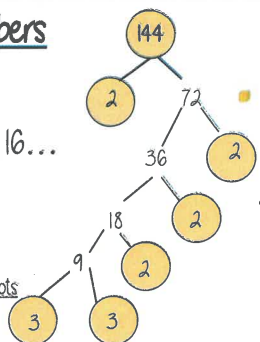
$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$(2 \times 2 \times 3) \times (2 \times 2 \times 3)$$

12 x 12

Prime factors can find square roots

$$\sqrt{144} = 12$$



#### Cube numbers

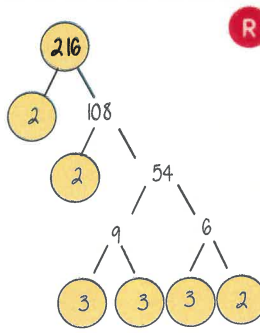
1, 8, 27, 64, 125...

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$(2 \times 3) \times (2 \times 3) \times (2 \times 3)$$

6 x 6 x 6

$$\sqrt[3]{216} = 6$$



### Higher powers and roots

$x^n$  ← n – power (number of times multiplied by itself)

x – the base number

$\sqrt[n]{x}$  ← Finding the nth root of any value

Other mental strategies for square roots

$$\begin{aligned} \sqrt{810000} &= \sqrt{81} \times \sqrt{10000} \\ &= 9 \times 100 \\ &= 900 \end{aligned}$$

### Standard form

Any number between 1 and less than 10

$$A \times 10^n$$

Any integer

0.001

$$1 \times \frac{1}{1000}$$

$$1 \times 10^{-3}$$

|        |        |                |                 |                  |
|--------|--------|----------------|-----------------|------------------|
| 10     | 1      | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ |
| $10^1$ | $10^0$ | $10^{-1}$      | $10^{-2}$       | $10^{-3}$        |
| 10     | 1      | 0.1            | 0.01            | 0.001            |

Any value to the power 0 always = 1

Numbers in standard form with negative powers will be less than 1

$$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$$

Negative powers do not indicate negative solutions

#### Example

$$3.2 \times 10^4$$

$$= 3.2 \times 10 \times 10 \times 10 \times 10$$

$$= 32000$$

#### Non-example

$$0.8 \times 10^4$$

$$5.3 \times 10^{07}$$

### Addition/ Subtraction Laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

### Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$

$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

$$\begin{aligned} 2^2 &= 4 \\ 2^1 &= 2 \\ 2^0 &= 1 \\ 2^{-1} &= \frac{1}{2} \\ 2^{-2} &= \frac{1}{4} \end{aligned}$$

Looking at the sequence can help to understand negative powers

### Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$$

The same base and power is repeated Use the addition law for indices

$$(2^3)^4 = 2^{12} \leftarrow a \times b = 3 \times 4 = 12$$

NOTICE the difference

$$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

### Standard form calculations

#### Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

#### Method 1

$$= 600000 + 800000$$

$$= 1400000$$

$$= 1.4 \times 10^6$$

#### Multiplication and division

$$1.5 \times 10^5$$

$$0.3 \times 10^3$$

$$(1.5 \times 10^5) \div (0.3 \times 10^3)$$

$$(1.5 \div 0.3) \times 10^5 \div 10^3$$

$$= 5 \times 10^2$$

#### Method 2

$$= (6 + 8) \times 10^5$$

$$= 14 \times 10^5$$

$$= 1.4 \times 10^1 \times 10^5$$

$$= 1.4 \times 10^6$$

This is not the final answer

Division questions can look like this

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations





# PROPORTION...

## Percentages and Interest

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Convert and compare FDP
- Work out percentages of amounts
- Increase/ decrease by a given percentage
- Express one number as a percentage
- Calculate simple and compound interest
- Calculate repeated percentage change
- Find the original value
- Solve problems with growth and decay

### Keywords

**Exponent:** how many times we use a number in multiplication. It is written as a power

**Compound interest:** calculating interest on both the amount plus previous interest

**Depreciation:** a decrease in the value of something over time.

**Growth:** where a value increases in proportion to its current value such as doubling

**Decay:** the process of reducing an amount by a consistent percentage rate over time.

**Multiplier:** the number you are multiplying by

**Equivalent:** of equal value.

### Compare FDP

R

Comparisons are easier in the same format

70/100 → This also means 70 ÷ 100 → 70 out of 100 squares → 70 "hundredths" = 7 "tenths" = 0.7 → 70 hundredths = 70% →

Using a calculator → →  $\frac{70}{100}$  → Convert to a decimal →  $\times 100$  converts to a percentage

This will give you the answer in the simplest form

Be careful of recurring decimals  
eg  $\frac{1}{3} = 0.333333$   
 $\frac{2}{3} = 0.\dot{6}$   
The dot above the 3

### Fraction/ Percentage of amount

R

Find  $\frac{3}{5}$  of £60 →

Remember  $\frac{3}{5} = 60\%$

10% of £60 = £6  
50% of £60 = £30  
60% of £60 = £36

Remember  $\frac{3}{5} = 60\% = 0.6$   
60% of £60 =  $0.6 \times 60 = £36$

### Percentage increase/decrease

R

Decrease by 58% →

100% - 58% = 42%  
100 - 58 = 42

Multiplier Less than 1

Increase by 12% →

100% + 12% = 112%  
100 + 12 = 112

Multiplier More than 1

### Express as a percentage

R

27 per every 50 shaded →  $\frac{27}{50}$

54 per every 100 shaded →  $\frac{54}{100}$  = 54%

$\frac{13}{30}$  → →  $\frac{13}{30} \times 100$  → 43.333...% → 43%

Can't use equivalence easily to find 'per hundred'

Decimal percentages are still a percentage.

### Simple and compound interest

**Simple Interest**

James invests £2000 at 5% simple interest →

The original value increases by this amount every year

**Compound Interest**

Tess invests £100 at 10% compound interest for 3 years

Original amount: £100

Y1: £110 (Multiplier 1.10)

Y2: £121 (Multiplier 1.10)

Y3: £132.10 (Multiplier 1.10)

The multiplier 1.10 repeats each year

### Repeated percentage change

**Compound Interest**

Tess invests £100 at 10% compound interest for 3 years

Original amount: £100

Repeated multiplier:  $\times 1.10$

Number of occurrences: 3

The multiplier:  $\times 1.10 \times 1.10 \times 1.10 = \times 1.331$

**Depreciation**

Depreciation calculations use multipliers less than 1

Multipliers are commutative - an overall multiplier effect can be calculated by combining the multipliers separately.

eg Increase of 10% then a reduction of 10% →  $\times 1.10 \times 0.9 = \times 0.99$

### Growth and decay

**Compound growth** →

**Compound decay** →

Compound growth and compound decay are exponential graphs

**Decay** - the values get closer to 0  
The constant multiplier is less than one

**Growth** - the values increase exponentially  
The constant multiplier is more than one

### Find the original value

Percentage calculations

Original amount  $\times$  Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test?

Original  $\times 0.6 = 24$

$24 \div 0.6 = 40$  marks

10% = 6  
100% = 40

Total questions on test

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?

Original  $\times 1.2 = 3000$

120% = £3000  
10% = £250  
100% = £2500

# PROPORTION...

## Ratios and fractions

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Compare quantities using ratio
- Link ratios and fractions and make comparisons
- Share in a given ratio
- Link Ratio and scales and graphs
- Solve problems with currency conversions
- Solve 'best buy' problems
- Combine ratios

### Keywords

**Ratio:** a statement of how two numbers compare

**Equivalent:** of equal value

**Proportion:** a statement that links two ratios

**Integer:** whole number, can be positive, negative or zero.

**Fraction:** represents how many parts of a whole.

**Denominator:** the number below the line on a fraction. The number represent the total number of parts.

**Numerator:** the number above the line on a fraction. The top number. Represents how many parts are taken

**Origin:** (0,0) on a graph. The point the two axes cross

**Gradient:** The steepness of a line

### Compare with ratio R

"For every dog there are 2 cats"



The ratio has to be written in the same order as the information is given

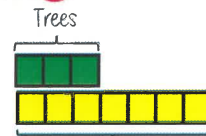
eg 2:1 would represent 2 dogs for every 1 cat

Units have to be of the same value to compare ratios

### Ratios and fraction R

Trees: Flowers

3:7



Ratio

Fraction of trees

Number of parts of in group  
Total number of parts

3  
10

Fraction

### Sharing a whole into a given R

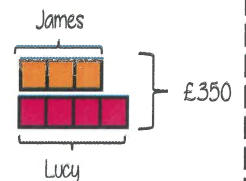
ratio

James and Lucy share £350 in the ratio 3:4  
Work out how much each person earns

Model the Question

James: Lucy

3:4



Find the value of one part

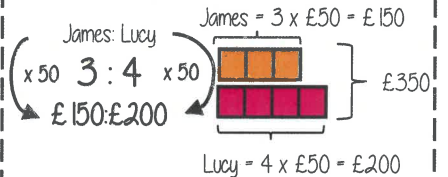
Whole: £350

7 parts to share between  
(3 James, 4 Lucy)

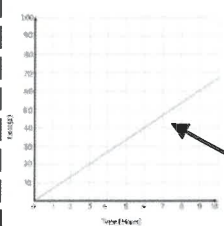
$£350 \div 7 = £50$

□ = one part  
= £50

Put back into the question



### Ratio and graphs R



Graphs with a constant ratio are directly proportional

- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

### Ratio and scale R

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image: Real life  
10cm: 300cm



### Conversion between currencies R

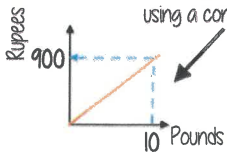
£1 = 90 Rupees

Currency is directly proportional

For every £1 I have 90 Rupees

$\times 10$  £1 = 90 Rupees  
 $\times 10$  £10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

$\times 7$  £1 = 90 Rupees  
 $\times 7$  £7 = 630 Rupees

### Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit. Therefore Divide by 4

4:20  
1:5

This side has to be divided by 4 too - to keep in proportion

the n part does not have to be an integer for this type of question

### Best buys



4 pens costs £2.60

£2.60 ÷ 4 = £0.65

4 ÷ 2.60 = 1.54 pens



10 pens costs £6.00

£6.00 ÷ 10 = £0.60

10 ÷ 6 = 1.67 pens

You could work out how much 40 pens are and then compare

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

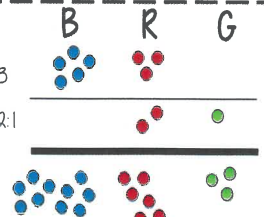
The best value means £1 buys you more pens

### Combining ratios

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green



10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share



# REASONING WITH ALGEBRA...

## Testing conjectures

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

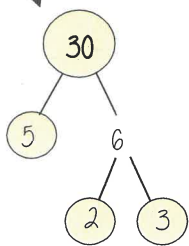
- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

### Keywords

- Multiples:** found by multiplying any number by positive integers
- Factor:** integers that multiply together to get another number.
- Prime:** an integer with only 2 factors.
- HCF:** highest common factor (biggest factor two or more numbers share)
- LCM:** lowest common multiple (the first time the times table of two or more numbers match)
- Verify:** the process of making sure a solution is correct
- Proof:** logical mathematical arguments used to show the truth of a statement
- Binomial:** a polynomial with two terms
- Quadratic:** a polynomial with four terms (often simplified to three terms)

### Factors, Multiples and Primes

Multiplication part-whole models



All three prime factor trees represent the same decomposition

**HCF – Highest common factor**

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors are factors two or more numbers share

**LCM – Lowest common multiple**

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

Common multiples are multiples two or more numbers share



### True or False?

**Conjecture**

A pattern that is noticed for many cases

1, 2, 4, ...  
The numbers in the sequence are doubling each time.

**Counterexamples**



This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

### Always, Sometimes, Never true.

**Always** Every value always supports the statement

**Sometimes** Examples show the statement being true and counter examples to show when it is false.

**Never** No example supports the statement

**Examples to try**

- 0 and 1
- Fractions
- Negative numbers

### Show that

**Numerical verification**

Show the stages to a solution with numerical values

**Algebraic verification**

Show algebraic properties of the solution  
You may want to use pictorial images to support this

**Proof**

Simple proofs using algebra

Compare the left hand side of an equation with the right hand side – are they the same or different?

### Conjectures



**Even**  
(2n)

Multiple of 2



**Odd**  
(2n + 1)

One more than any even

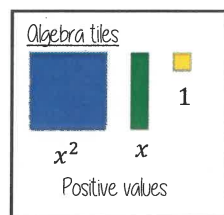
Use numerical verification first  
Use pictorial verification – the representations of numbers of odd and even

### Expanding binomials

$$2(x + 2) \equiv 2x + 4$$

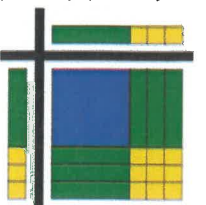


Algebra tiles can represent a binomial expansion  
Has two terms



The order of the binomial has no impact on the outcome.  
eg (x + 3)(3 + x)

$$(x + 3)(x + 3) \equiv x^2 + 6x + 9$$



This is a quadratic  
It has four terms which simplified to three terms

### Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers

Positions of numbers in relation to n form expressions.

Eg one space to the right of n  
n + 1

Eg One row below n  
n + 10

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

The size of the grid for generalisation changes the relationship statements

# DEVELOPING ALGEBRA...

## Representing solutions of equations and inequalities

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve equations and inequalities
- Represent and interpret solutions on a number line as inequalities
- Draw straight line graphs and find solutions to equations
- Form and solve equations and inequalities with unknowns on both sides

### Keywords

**Solution:** a value we can put in place of a variable that makes the equation true

**Variable:** a symbol for a number we don't know yet

**Equation:** an equation says that two things are equal – it will have an equals sign =

**Expression:** numbers, symbols and operators grouped together to show the value of something

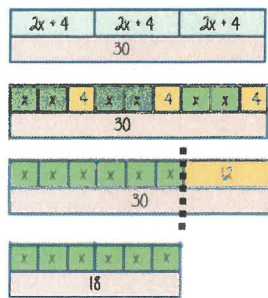
**Identity:** An equation where both sides have variables that cause the same answer includes  $\equiv$

**Linear:** an equation or function that is the equation of a straight line

**Intersection:** the point that two lines meet

**Inequality:** an inequality compares two values showing if one is greater than, less than or equal to another.

### Solve equations R



$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$-12 \quad -12$$

$$6x = 18$$

$$-6 \quad +6$$

$$x = 3$$

Substitute to check your answer. This could be negative or a fraction or decimal

### Form and solve inequalities R



Two more than treble my number is greater than 11

Form

$$x \rightarrow x3 \rightarrow +2 \rightarrow 11$$

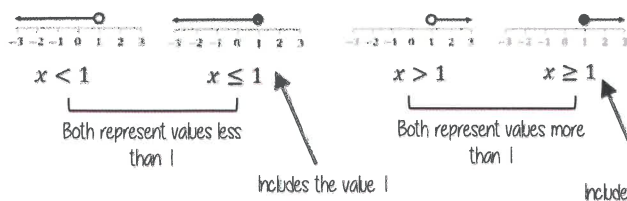
$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

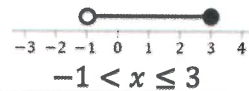
$$x > 3$$

### Solutions on a number line



- Includes the value it sits above
- Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1



This includes the integer values 0, 1, 2, 3

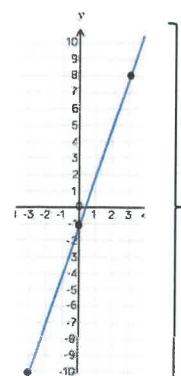
### Plotting straight line graphs R

$$y = 3x - 1$$

|   |     |    |   |
|---|-----|----|---|
| x | -3  | 0  | 3 |
| y | -10 | -1 | 8 |

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

### Find solutions graphically

For linear equations there is only one point the graph meets the x value

$$x = 2$$

$$y = 4$$

These two lines will cross at (2,4) because they are just x- and y- they are parallel to axes and meet in one place

$$y = 3x - 1$$

$$y = 8$$

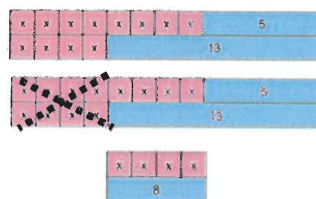
$$3x - 1 = 8$$

Remember equation of a line format is  $y = mx + c$

The solution is the point the two lines meet (3,8)

### Equations: unknown on both sides R

$$8x + 5 = 4x + 13$$



$$8x + 5 = 4x + 13$$

$$-4x \quad -4x$$

$$4x + 5 = 13$$

$$-5 \quad -5$$

$$4x = 8$$

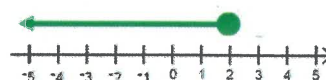
$$+4 \quad +4$$

$$x = 2$$

### inequalities: unknown on both sides

$$8x + 5 \leq 4x + 13$$

$$x \leq 2$$



Any value 2 or less will satisfy this inequality



# — DEVELOPING ALGEBRA... Simultaneous Equations

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether  $(x,y)$  is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

## Keywords

**Solution:** a value we can put in place of a variable that makes the equation true

**Variable:** a symbol for a number we don't know yet

**Equation:** an equation says that two things are equal — it will have an equals sign =

**Substitute:** replace a variable with a numerical value

**LCM:** lowest common multiple (the first time the times table of two or more numbers match)

**Eliminate:** to remove

**Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

**Coordinate:** a set of values that show an exact position

**Intersection:** the point two lines cross or meet

## Is $(x, y)$ a solution?

$x$  and  $y$  represent values that can be substituted into an equation

Does the coordinate  $(1,8)$  lie on the line  $y=3x+5$ ?

This coordinate represents  $x=1$  and  $y=8$

$$y = 3x + 5$$

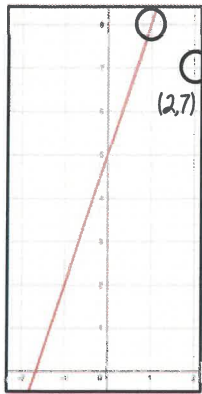
$$8 = 3(1) + 5$$

As the substitution makes the equation correct the coordinate  $(1,8)$  IS on the line  $y=3x+5$

Is  $(2,7)$  on the same line?

$$7 \neq 3(2) + 5$$

No 7 does NOT equal 6+5



## Substituting known variables

A line has the equation  $3x + y = 14$

Two different variables, two solutions

Stephanie knows the point  $x = 4$  lies on that line. Find the value for  $y$ .

$$x = 4$$

$$3x + y = 14$$

$$3(4) + y = 14$$

$$12 + y = 14$$

$$-12 \quad -12$$

$$y = 2$$



## Substituting in an expression

Substitute  $2y$  in place of the  $x$  variable as they represent the same value

$$x = 2y$$

$$x + y = 30$$



$$x = 2y \quad x + y = 30$$

$$3y = 30$$

$$\div 3 \quad \div 3$$

$$y = 10$$

$$x = 2y$$

$$x = 20$$

Pair of simultaneous equations (two representations)

## Solve graphically

$$x + y = 6$$

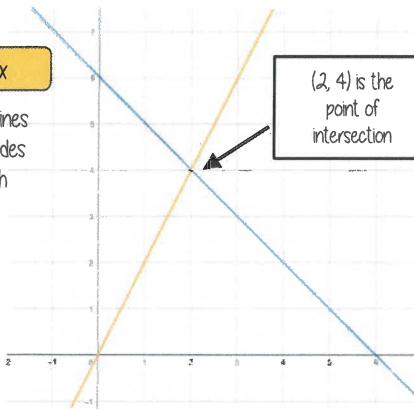
$$y = 2x$$

$(2, 4)$  is the point of intersection

Linear equations are straight lines. The point of intersection provides the  $x$  and  $y$  solution for both equations

The solution that satisfies both equations is

$$x = 2 \text{ and } y = 4$$



## Solve by subtraction

$$3x + 2y = 18$$

$$- \quad x + 2y = 10$$

$$3x + 2y = 18$$

$$- \quad x + 2y = 10$$

$$2x = 8$$

$$\div 2 \quad \div 2$$

$$x = 4$$

$$x + 2y = 10$$

$$(4) + 2y = 10$$

$$-4 \quad -4$$

$$2y = 6$$

$$\div 2 \quad \div 2$$

$$y = 3$$

$$x = 4$$

$$y = 3$$

$$3x + 2y = 18$$

$$- \quad x + 2y = 10$$

$$2x = 8$$

$$x + 2y = 10$$

$$(4) + 2y = 10$$

$$-4 \quad -4$$

$$2y = 6$$

$$\div 2 \quad \div 2$$

$$y = 3$$

## Solve by addition

Addition makes zero pairs

$$3x + 2y = 16$$

$$+ \quad 6x - 2y = 2$$

$$9x = 18$$

$$\div 9 \quad \div 9$$

$$x = 2$$

$$3x + 2y = 16$$

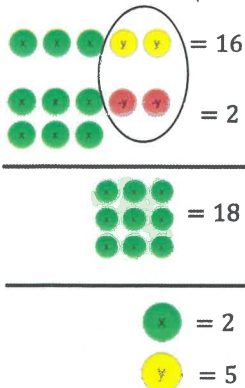
$$3(2) + 2(y) = 16$$

$$6 + 2y = 16$$

$$-6 \quad -6$$

$$2y = 10$$

$$y = 5$$



## Solve by adjusting one

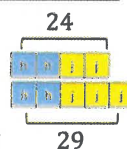
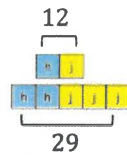
$$h + j = 12 \quad \text{No equivalent values}$$

$$2h + 2j = 29$$

$$2h + 2j = 24$$

$$2h + 2j = 29$$

By proportionally adjusting one of the equations — now solve the simultaneous equations choosing an addition or subtraction method



## Solve by adjusting both

$$2x + 3y = 39$$

$$5x - 2y = -7$$

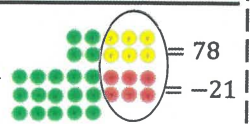
Use LCM to make equivalent  $x$  OR  $y$  values. Because of the negative values using zero pairs and  $y$  values is chosen choice

$$4x + 6y = 78$$

$$15x - 6y = -21$$

Now solve by addition

Addition makes zero pairs



# REASONING WITH ALGEBRA... Straight Line Graphs

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use  $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

## Keywords

**Gradient:** the steepness of a line

**Intercept:** where two lines cross. The y-intercept: where the line meets the y-axis.

**Parallel:** two lines that never meet with the same gradient.

**Co-ordinate:** a set of values that show an exact position on a graph

**Linear:** linear graphs (straight line) – linear common difference by addition/ subtraction

**Asymptote:** a straight line that a graph will never meet.

**Reciprocal:** a pair of numbers that multiply together to give 1.

**Perpendicular:** two lines that meet at a right angle.

## Lines parallel to the axes

R

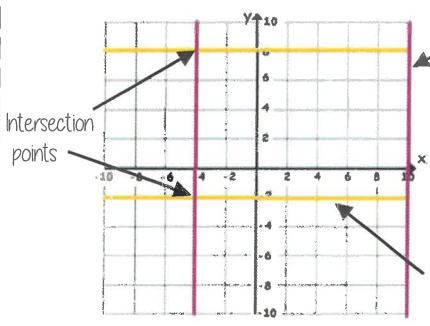
All the points on this line have a x coordinate of 10

Lines parallel to the y axis take the form  $x = a$  and are vertical

Lines parallel to the x axis take the form  $y = a$  and are horizontal

All the points on this line have a y coordinate of -2 e.g. (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

'a' can be ANY positive or negative value including 0



## Plotting $y = mx + c$ graphs

R

$y = 3x - 1$  → 3 x the x coordinate then - 1

|   |     |    |   |
|---|-----|----|---|
| x | -3  | 0  | 3 |
| y | -10 | -1 | 8 |

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

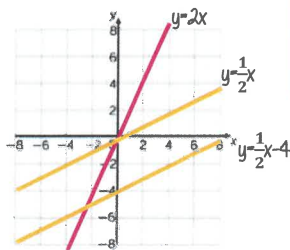
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

## Compare Gradients

$$y = mx + c$$

The coefficient of x (the number in front of x) tells us the gradient of the line



The greater the gradient – the steeper the line

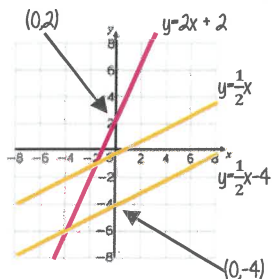
Positive gradients

Parallel lines have the same gradient

Negative gradients

## Compare Intercepts

$y = mx + c$  ← The value of c is the point at which the line crosses the y-axis. Y intercept



The coordinate of a y intercept will always be (0,c)

Lines with the same y-intercept cross in the same place

$$y = mx + c$$

The coefficient of x (the number in front of x) tells us the gradient of the line

$y = mx + c$  ← The value of c is the point at which the line crosses the y-axis. Y intercept  
y and x are coordinates

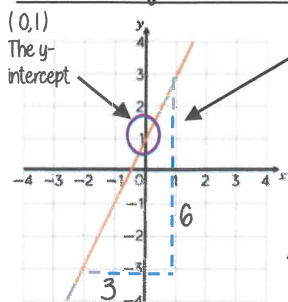
The equation of a line can be rearranged. Eg

$$y = c + mx$$

$$c = y - mx$$

Identify which coefficient you are identifying or comparing

## Find the equation from a graph



The Gradient  $\frac{6}{3} = 2$

$$y = 2x + 1$$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

## Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

|          |     |   |   |   |      |
|----------|-----|---|---|---|------|
| Time (h) | 0   | 1 | 2 | 3 | 8    |
| Cost (£) | £25 |   |   |   | £125 |

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

## Direct Proportion graphs

To represent direct proportion the graph must start at the origin

When you have 0 pens this has 0 cost. The gradient shows the price per pen

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

|          |   |       |   |   |   |
|----------|---|-------|---|---|---|
| Boxes    | 0 | 1     | 2 | 3 | 8 |
| Cost (£) |   | £2.30 |   |   |   |

The y-intercept shows the minimum charge. The gradient represents the price per mile



# PROPORTION...

# Probability

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Add, Subtract and multiply fractions
- Find probabilities using likely outcomes
- Use probability that sums to 1
- Estimate probabilities
- Use Venn diagrams and frequency trees
- Use sample space diagrams
- Calculate probability for independent events
- Use tree diagrams

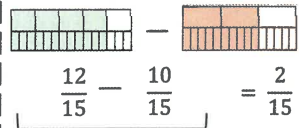
## Keywords

- Event:** one or more outcomes from an experiment
- Outcome:** the result of an experiment
- Intersection:** elements (parts) that are common to both sets
- Union:** the combination of elements in two sets
- Expected Value:** the value/ outcome that a prediction would suggest you will get
- Universal Set:** the set that has all the elements
- Systematic:** ordering values or outcomes with a strategy and sequence
- Product:** the answer when two or more values are multiplied together.

## Add, Subtract and multiply fractions

Addition and Subtraction

$$\frac{4}{5} - \frac{2}{3}$$



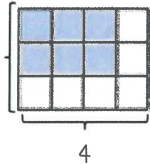
Use equivalent fractions to find a common multiple for both denominators

Multiplication

$$\frac{3}{4} \times \frac{2}{3}$$

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Modelled:



Total number of parts in the diagram

## Likelihood of a probability

Impossible 0 or 0%      Even chance 0.5, 1/2 or 50%      Certain 1 or 100%

The more likely an event the further up the probability it will be in comparison to another event. (It will have a probability closer to 1)

## Sum to 1

Probability is always a value between 0 and 1

The probability of getting a blue ball is  $\frac{1}{5}$   
 $\therefore$  The probability of NOT getting a blue ball is  $\frac{4}{5}$

The sum of the probabilities is 1

## Experimental data

Theoretical probability

What we expect to happen

Experimental probability

What actually happens when we try it out

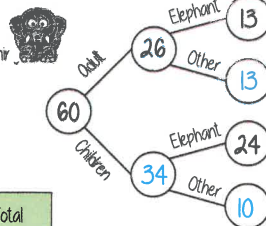
The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.  
 Theoretical probability is proportional

## Tables, Venn diagrams, Frequency trees

### Frequency trees

60 people visited the zoo one Saturday morning. 13 of the adults' favourite animal was an elephant. 24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

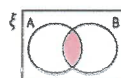
$$P(\text{adult}) = \frac{26}{60}$$

$$P(\text{Child with favourite animal as elephant}) = \frac{13}{37}$$

### Two-way table

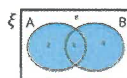
|          | Adult | Child | Total |
|----------|-------|-------|-------|
| Elephant | 13    | 24    | 37    |
| Other    | 13    | 10    | 23    |
| Total    | 26    | 34    | 60    |

### Venn diagram



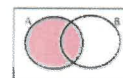
in set A AND set B

$$P(A \cap B)$$



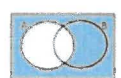
in set A OR set B

$$P(A \cup B)$$



in set A

$$P(A)$$



NOT in set A

$$P(A')$$

## Sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

|   | 1  | 2  | 3  | 4  | 5  | 6  |
|---|----|----|----|----|----|----|
| H | 1H | 2H | 3H | 4H | 5H | 6H |
| T | 1T | 2T | 3T | 4T | 5T | 6T |

$$P(\text{Even number and tails}) = \frac{3}{12}$$

## Independent events

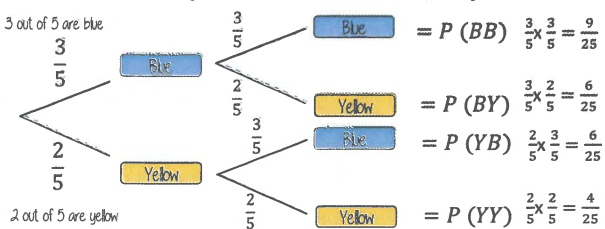
The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

$$P(A \text{ and } B) = P(A) \times P(B)$$

### Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick.

Because they are replaced the second pick has the same probability

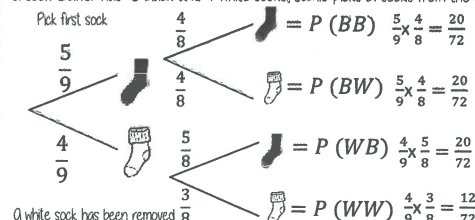


## Dependent events

### Tree diagram for dependent event

The outcome of the first event has an impact on the second event

A sock drawer has 5 black and 4 white socks. Jamie picks 2 socks from the drawer.



NOTE: as 'socks' are removed from the drawer the number of items in that drawer is also reduced.  $\therefore$  the denominator is also reduced for the second pick.

# DELVING INTO DATA...

## Collecting, representing and interpreting

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Construct and interpret frequency tables and polygon two-way tables, line, bar, & pie charts
- Find and interpret averages from a list and a table
- Construct and interpret time series graphs, stem and leaf diagrams and scatter graphs

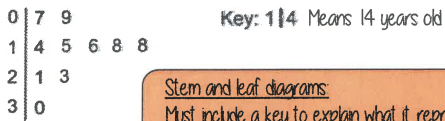
### Keywords

- Population:** the whole group that is being studied
- Sample:** a selection taken from the population that will let you find out information about the larger group
- Representative:** a sample group that accurately represents the population
- Random sample:** a group completely chosen by chance. No predictability to who it will include.
- Bias:** a built-in error that makes all values wrong by a certain amount
- Primary data:** data collected from an original source for a purpose.
- Secondary data:** data taken from an external location. Not collected directly.
- Outlier:** a value that stands apart from the data set

### Stem and leaf

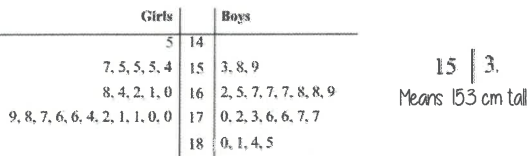
A way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket



**Stem and leaf diagrams:**  
Must include a key to explain what it represents  
The information in the diagram should be ordered

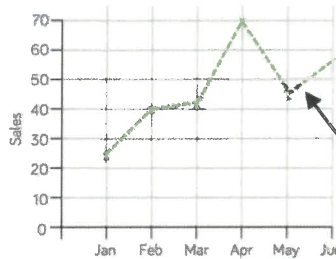
Back to back stem and leaf diagrams



**Back to back stem and leaf diagrams:**  
Allow comparisons of similar groups  
Allow representations of two sets of data

### Time-Series

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

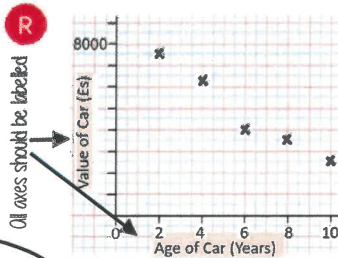
### Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

- Mean, mode, median — allows for a comparison about more or less average.
- Range — allows for a comparison about reliability and consistency of data

### Draw and interpret a scatter graph

| Age of Car (Years) | 2    | 4    | 6    | 8    | 10   |
|--------------------|------|------|------|------|------|
| Value of Car (£s)  | 7500 | 6250 | 4000 | 3500 | 2500 |



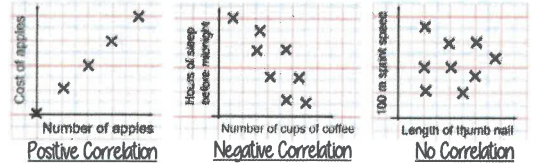
- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

"This scatter graph shows as the age of a car increases the value decreases"

The link between the data can be explained verbally

The axis should fit all the values on and be equally spread out

### Linear Correlation



As one variable increases so does the other variable

As one variable increases the other variable decreases

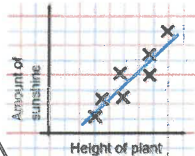
There is no relationship between the two variables

### The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph

#### Things to know:

- The line of best fit **DOES NOT** need to go through the origin (the point the axes cross)
- There should be approximately the same number of points above and below the line (it may not go through any points)
- The line extends across the whole graph



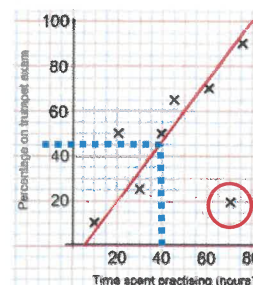
It is only an estimate because the line is designed to be an average representation of the data

It is always a **straight line**.

### Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45.



Extrapolation is where we use our line of best fit to predict information outside of our data

\*\*This is not always useful — in this example you cannot score more than 100%. So revising for longer can not be estimated\*\*

This point is an "outlier" it is an outlier because it doesn't fit this model and stands apart from the data



# - DELVING INTO DATA...

## Collecting, representing and interpreting

@whisto\_maths

### What do I need to be able to do?

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### Keywords

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**Outlier:** a value that stands apart from the data set

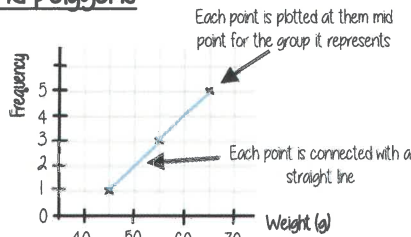
### Frequency tables and polygons

| x<br>Weight(g)   | Frequency |
|------------------|-----------|
| $40 < x \leq 50$ | 1         |
| $50 < x \leq 60$ | 3         |
| $60 < x \leq 70$ | 5         |

We do not know from grouped data where each value is placed so have to use an estimate for calculations

#### MID POINTS

Mid-points are used as estimated values for grouped data. The middle of each group

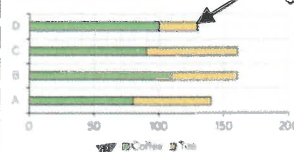


The data about weight starts at 40. So the axis can start at 40

Mid-point  
Start point + End point  
2

### Bar and line charts

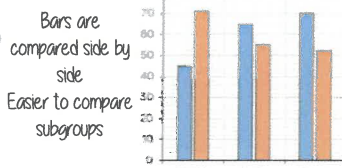
#### Composite bar charts



Categories clearly indicated

Compare the bars green compared to yellow. The size of each bar is the frequency. Overall total easily comparable.

#### Dual bar charts



Categories clearly indicated

Bars are compared side by side. Easier to compare subgroups

### Averages from a table

#### Non-grouped data

|                    |   |   |    |
|--------------------|---|---|----|
| Number of Siblings | 0 | 1 | 2  |
| Frequency          | 6 | 8 | 6  |
| Subtotal           | 0 | 8 | 12 |

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean:  $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

#### Grouped data

| x<br>Weight(g)   | Frequency | Mid Point | MP x Freq |
|------------------|-----------|-----------|-----------|
| $40 < x \leq 50$ | 1         | 45        | 45        |
| $50 < x \leq 60$ | 3         | 55        | 165       |
| $60 < x \leq 70$ | 5         | 65        | 325       |

Overall Frequency: 9

Overall Total : 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

### Two way tables

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adults favourite animal was an elephant. 24 of the children's favourite animal was an elephant.

Extract information to input to the two-way table.

|          | Adult | Child | Total |
|----------|-------|-------|-------|
| Elephant | 13    | 24    | 37    |
| Other    | 13    | 10    | 23    |
| Total    | 26    | 34    | 60    |

Subgroups each have their own heading

Needs subgroup totals

Overall total

### Draw and interpret Pie Charts

| Type of pet | Dog | Cat | Hamster |
|-------------|-----|-----|---------|
| Frequency   | 32  | 25  | 3       |

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$  "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192^\circ$



Use a protractor to draw. This is  $192^\circ$

Multiple method  
As 60 goes into 360 - 6 times.  
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Comparing Pie Charts:  
You NEED the overall frequency to make any comparisons

### Averages from lists

#### The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values)

55

Divide the overall total by how many pieces of data you have

$55 \div 5$

Mean = 11

#### The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

This can still be easier if the data is ordered first

Mode = 8

#### The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8

Put the data in order

4, 8, 8, 11, 24

Find the value in the middle

4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

#### For Grouped Data

The modal group - which group has the highest frequency

# - GEOMETRY...

## Angles and bearings

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent bearings
- Measure and read bearings
- Make scale drawings using bearings
- Calculate bearings using angle rules
- Solve bearings problems using Pythagoras and trigonometry

### Keywords

**Cardinal directions:** the directions of North, South, East, West

**Angle:** the amount of turn between two lines around their common point

**Bearing:** the angle in degrees measured clockwise from North.

**Perpendicular:** where two lines meet at  $90^\circ$

**Parallel:** straight lines always the same distance apart and never touch. They have the same gradient

**Clockwise:** moving in the direction of the hands on a clock.

**Construct:** to draw accurately using a compass, protractor and or ruler or straight edge.

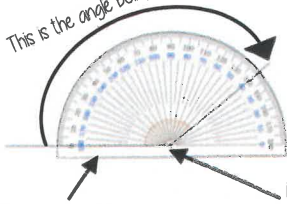
**Scale:** the ratio of the length of a drawing to the length of the real thing

**Protractor:** an instrument used in measuring or drawing angles.

### Measure angles to $180^\circ$

**R**

This is the angle being measured



Read from  $0^\circ$  on the base line. Remember to use estimation. This is an obtuse angle so between  $90^\circ$  and  $180^\circ$

The base line follows the line segment

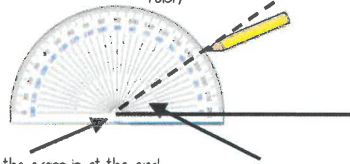
Make sure the cross is at the point the two lines meet

### Draw angles up to $180^\circ$

**R**

Draw a  $35^\circ$  angle

Make a mark at  $35^\circ$  with a pencil. And join to the angle point (use a ruler)

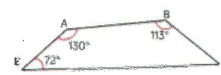


Make sure the cross is at the end of the line (where you want the angle)

The angle

### Angle notation

The letter in the middle is the angle. The arc represents the part of the angle.



Angle Notation: three letters **ABC**. This is the angle at  $B = 113^\circ$ .  $\angle ABC$  is also used to represent the angle at B.

### Scale drawings

**R**

1:20

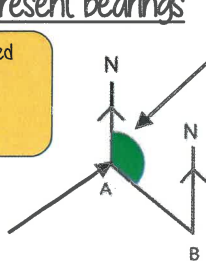
For every 1cm on the model there are 20cm in real life.

Remember: Scale drawings **ONLY** change lengths and distances. Angles remain the same.

### Understand and represent bearings

- A bearing is always measured from **NORTH**
- It is always given as three figures

The bearing of B from A is calculated by measuring the highlighted angle.



The angle indicated starts from the North line at A and joins the path connecting A to B.

This angle shows the bearing of B from A.

The sentence... "Bearing of \_\_\_ from \_\_\_" is really important in identifying the bearing being represented.

Using **estimation** it is clear this angle is between  $090^\circ$  and  $180^\circ$

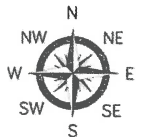
### Directions



Clockwise

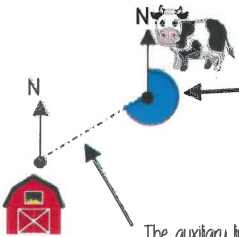


Anti-Clockwise



### Measure and read bearings

The bearing of the cow to the barn.

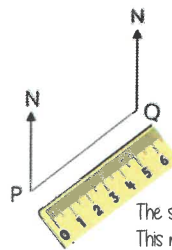


This angle is measured from **NORTH**. It is measured in a clockwise direction. **Estimation** indicates this angle is between  $180^\circ$  and  $270^\circ$ . Use a protractor to measure accurately. Remember: bearings are written as three figures.

The auxiliary line is drawn to help you measure and draw the angle that is measured to represent the bearing.

### Scale drawings using bearings

Remember - angles **DO NOT** change size in scaled drawings.



The bearing measurements do not change from "real life" to images.

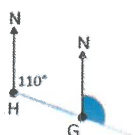
The scale may need to be calculated from the image. This represents 30km from P to Q.

The units in the ratio scale are the same.

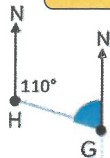
6cm = 30km  
6:3,000,000

### Bearings with angle rules

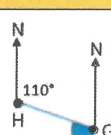
Because two North lines are **PARALLEL**...



They form **corresponding angles** and therefore are the same size.



They form **co-interior angles** and add up to  $180^\circ$ .



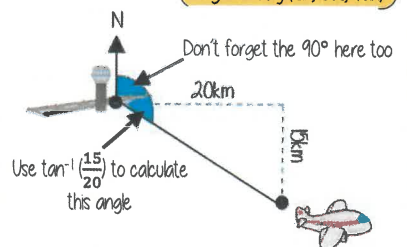
They form **alternate angles** and therefore are the same size.

### Bearings with right-angled geometry

Look for Right-angles. Pythagoras. Trigonometry (Sin, Cos, Tan)

"Due West" bearing of  $270^\circ$  makes a  $90^\circ$  angle. "Due East" bearing of  $090^\circ$  makes a  $90^\circ$  angle.

A plane flies East for 20km then turns South for 15km. Find the bearing of the plane from where it took off.



Use  $\tan^{-1}(\frac{15}{20})$  to calculate this angle.



# - CONSTRUCTING IN 2D/3D...

## Constructions & congruency

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

### Keywords

- Protractor:** piece of equipment used to measure and draw angles
- Locus:** set of points with a common property
- Equidistant:** the same distance
- Discorrectangle:** (a stadium) – a rectangle with semi circles at either end
- Perpendicular:** lines that meet at  $90^\circ$
- Arc:** part of a curve
- Bisector:** a line that divides something into two equal parts
- Congruent:** the same shape and size

### Draw and measure angles

R

Draw a  $35^\circ$  angle

Make a mark at  $35^\circ$  with a pencil and join to the angle point (use a ruler)

The angle

Make sure the cross is at the end of the line (where you want the angle)

### Scale drawings

R

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life  
1cm : 30cm  
10cm : 300cm

### Locus of a distance from a point

All points are equidistant (the same distance) from the fixed point in the middle.

Equipment needed  
The radius is the distance from the fixed point

If the point is in the corner it can only make a quarter circle

### Locus of a distance from a straight line

All points are equidistant (the same distance) from line

Equipment needed  
The line is straight so a ruler is used for the straight lines parallel to your original line

The ends of the line are fixed points

### Locus equidistant from two points

Also a perpendicular bisector  
Because if the points are joined, this new line intersects it at a  $90^\circ$

Join the intersections with a ruler.  
All points on this line are equidistant from both points

Keep the compass the same size and draw two arcs from each point

### Construct a perpendicular from a point

Use a compass and draw an arc that cuts the line. Use the point to place the compass

Keep the compass the same distance and now use your new points to make new intersecting arcs

Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

### Locus of a distance from two lines

Also an angle bisector  
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

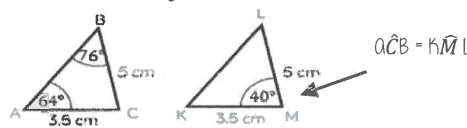
Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

### Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



Because all the angles are the same and  $AC = KM$   $BC = LM$  triangles  $ABC$  and  $KLM$  are congruent

### Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

### Constructing Triangles

Link to steps R

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side

# - SIMILARITY...

## Congruence, similarity & enlargement

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

### Keywords

**Enlarge:** to make a shape bigger (or smaller) by a given multiplier (scale factor)

**Scale Factor:** the multiplier of enlargement

**Centre of enlargement:** the point the shape is enlarged from

**Similar:** when one shape can become another with a reflection, rotation, enlargement or translation

**Congruent:** the same size and shape

**Corresponding:** items that appear in the same place in two similar situations

**Parallel:** straight lines that never meet (equal gradients)

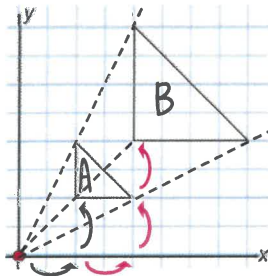
### Positive scale factors R

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

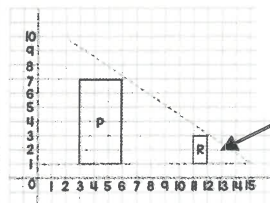
The distance from the point enlarges by 2



### Fractional scale factors R

Fractions less than 1 make a shape **SMALLER**

R is an enlargement of P by a scale factor  $\frac{1}{3}$  from centre of enlargement (15,1)



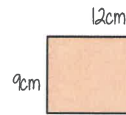
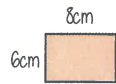
SF:  $\frac{1}{3}$  - R is three times smaller than P

### Identify similar shapes



Angles in similar shapes do not change.  
e.g. if a triangle gets bigger the angles can not go above  $180^\circ$

Similar shapes

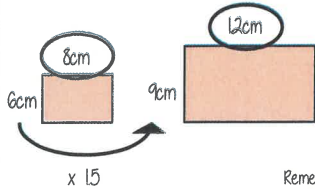


Scale Factor:  
Both sides on the bigger shape are 15 times bigger

Compare sides:  $\frac{6}{2} : \frac{8}{3}$

Both sets of sides are in the same ratio

### Information in similar shapes



Compare the equivalent side on both shapes

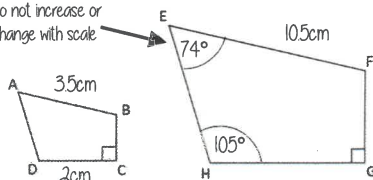
Scale Factor is the multiplicative relationship between the two lengths

Remember angles do not increase or change with scale

Shape ABCD and EFGH are similar

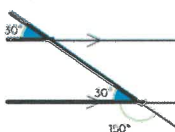
Notation helps us find the corresponding sides

AB and EF are corresponding



### Angles in parallel lines R

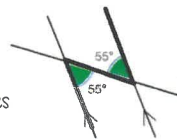
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

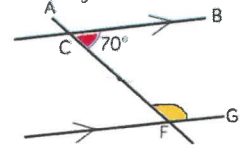
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

Because co-interior angles have a sum of  $180^\circ$  the highlighted angle is  $110^\circ$



As angles on a line add up to  $180^\circ$  co-interior angles can also be calculated from applying alternate/ corresponding rules first

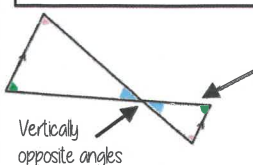
### Similar triangles

Shares a vertex

Because corresponding angles are equal the highlighted angles are the same size

Parallel lines - all angles will be the same in both triangle

As all angles are the same this is similar - it only one pair of sides are needed to show equality

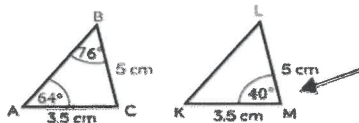


Vertically opposite angles

All the angles in both triangles are the same and so similar

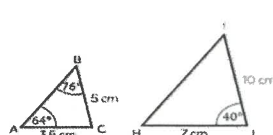
### Congruence and Similarity

Congruent shapes are identical - all corresponding sides and angles are the same size



$ABC = KLM$

Because all the angles are the same and  $AC = KM$   $BC = LM$  triangles ABC and KLM are congruent



Because all angles are the same, but all sides are enlarged by 2 ABC and HJ are similar

### Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

**Side-side-side**

All three sides on the triangle are the same size

**Angle-side-angle**

Two angles and the side connecting them are equal in two triangles

**Side-angle-side**

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

**Right angle-hypotenuse-side**

The triangles both have a right angle, the hypotenuse and one side are the same



# GEOMETRY...

## Working with circles

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere.
- Understand and use surface area of a cone, cylinder and sphere.

### Keywords

**Circumference:** the length around the outside of the circle – the perimeter

**Area:** the size of the 2D surface

**Diameter:** the distance from one side of a circle to another through the centre

**Radius:** the distance from the centre to the circumference of the circle

**Tangent:** a straight line that touches the circumference of a circle

**Chord:** a line segment connecting two points on the curve

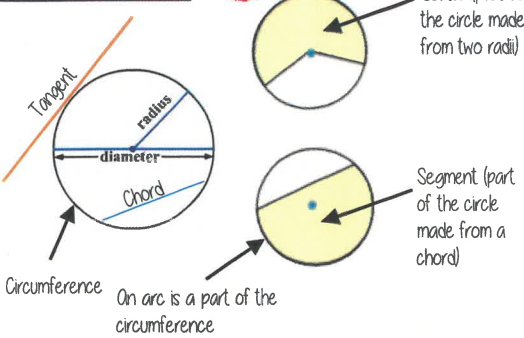
**Frustrum:** a pyramid or cone with the top cut off

**Hemisphere:** half a sphere

**Surface area:** the total area of the surface of a 3D shape.

### Parts of a circle

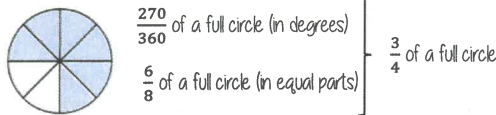
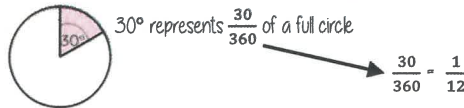
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### Fractional parts of a circle

A circle is made up of 360°

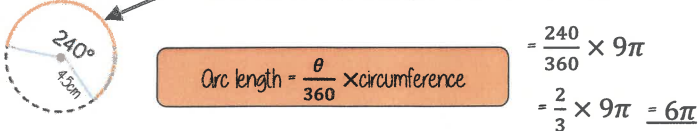
Formula to remember  
Area of a circle =  $\pi r^2$   
Circumference of a circle =  $\pi d$  or  $2\pi r$



The fraction of the circle is as  $\frac{\theta}{360}$   
 $\theta$  represents the degrees in the sector

### Arc length

Remember an arc is part of the circumference  
Circumference of the whole circle =  $\pi d = \pi \times 9 = 9\pi$



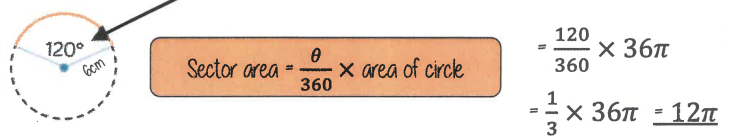
### Perimeter

Perimeter is the length around the outside of the shape  
This includes the arc length and the radii that encloses the shape

Perimeter =  $\frac{\theta}{360} \times \text{circumference} + 2r = 6\pi + 9$

### Sector area

Remember a sector is part of a circle  
Area of the whole circle =  $\pi r^2 = \pi \times 6^2 = 36\pi$



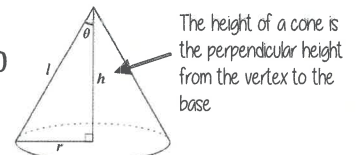
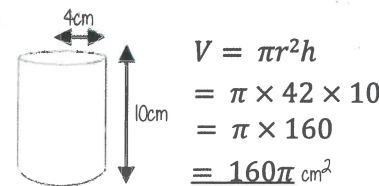
### Volume of a cone and a cylinder

Volume Cylinder =  $\pi r^2 h$

Volume Cone =  $\frac{1}{3} \pi r^2 h$

A cylinder is a prism – cross section is a circle

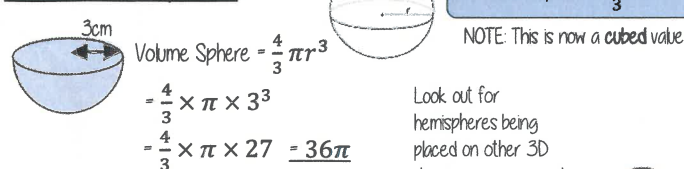
A cone is a pyramid with a circular base



Give your answer in terms of  $\pi$   
means NOT in terms of pi  $= 502.7 \text{ cm}^2$

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

### Volume of a sphere



Volume Sphere =  $\frac{4}{3} \pi r^3$

NOTE: This is now a cubed value

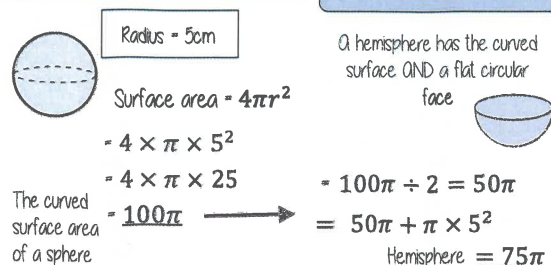
Look out for hemispheres being placed on other 3D shapes, e.g. cones and cylinders



A hemisphere is half the volume of the overall sphere  
 $= 36\pi \div 2 = 18\pi$

### Surface area of a sphere

Surface area =  $4\pi r^2$

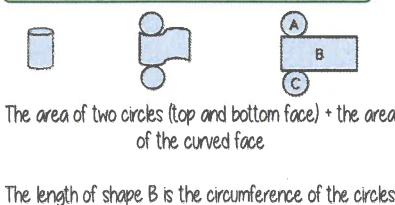


A hemisphere has the curved surface AND a flat circular face

### Surface area of cones and cylinders

Surface area cylinder =  $2\pi r^2 + \pi dh$

Curved surface area Cone =  $\pi rl$



Look out for the use of Pythagoras to calculate the length  $l$

Total surface area = curved face + circle face (area of base)

# — GEOMETRY...

# Vectors

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## What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

## Keywords

**Direction:** the line our course something is going

**Magnitude:** the magnitude of a vector is its length

**Scalar:** a single number used to represent the multiplier when working with vectors

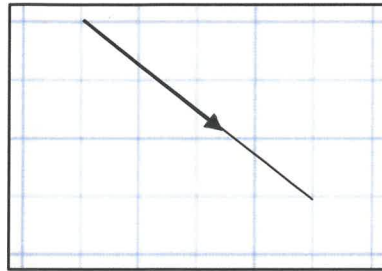
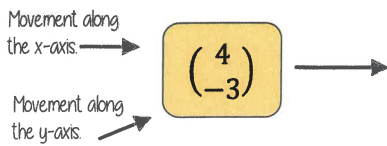
**Column vector:** a matrix of one column describing the movement from a point

**Resultant:** the vector that is the sum of two or more other vectors

**Parallel:** straight lines that never meet

## Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another



Vectors show both direction and magnitude

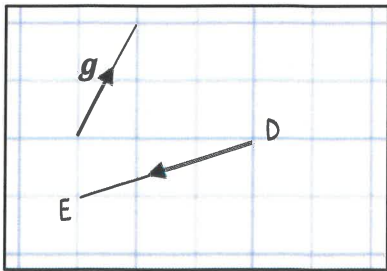
The arrow is pointing in the direction from starting point to end point of the vector.

The direction is important to correctly write the vector

The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)

The magnitude stays the same even if the direction changes

## Understand and represent vectors



Vector notation  $\overrightarrow{DE}$  is another way to represent the vector joining the point D to the point E

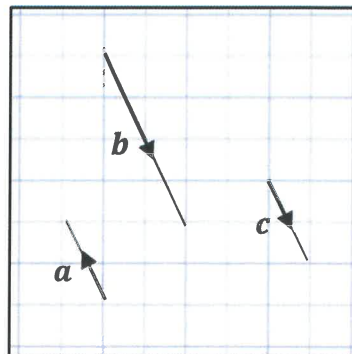
$$\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so  $\mathbf{g}$  represents the vector  $\mathbf{g} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

## Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$$\mathbf{b} = 2 \times \mathbf{c} = 2\mathbf{c}$$

Multiply  $\mathbf{c}$  by 2 this becomes  $\mathbf{b}$ . The two lines are parallel

$$\mathbf{a} = -1 \times \mathbf{c} = -\mathbf{c}$$

The vectors  $\mathbf{a}$  and  $\mathbf{c}$  are also parallel. A negative scalar causes the vector to reverse direction.

$$\mathbf{b} = -2 \times \mathbf{a} = -2\mathbf{a}$$

$$\mathbf{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

## Addition of vectors

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

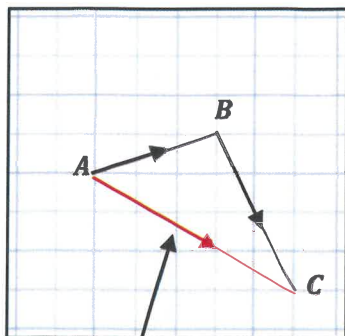
$$\overrightarrow{AB} + \overrightarrow{BC}$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 3+2 \\ 1+(-4) \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

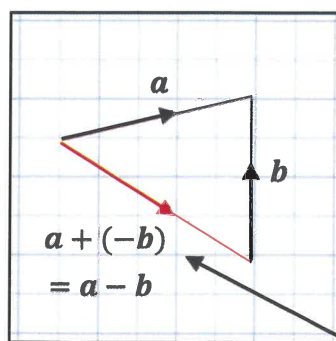
Look how this addition compares to the vector  $\overrightarrow{AC}$



The resultant

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

## Addition and subtraction of vectors



$$\mathbf{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$\mathbf{a} + (-\mathbf{b}) = \begin{pmatrix} 5+0 \\ 1+(-4) \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

$$\mathbf{a} + (-\mathbf{b}) = \mathbf{a} - \mathbf{b}$$

The resultant is  $\mathbf{a} - \mathbf{b}$  because the vector is in the opposite direction to  $\mathbf{b}$  which needs a scalar of  $-1$



# - SIMILARITY...

# Trigonometry

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## What do I need to be able to do?

By the end of this unit you should be able to:

- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

## Keywords

**Enlarge:** to make a shape bigger (or smaller) by a given multiplier (scale factor)

**Scale Factor:** the multiplier of enlargement

**Constant:** a value that remains the same

**Cosine ratio:** the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement

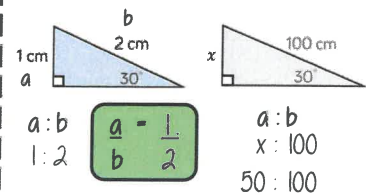
**Sine ratio:** the ratio of the length of the opposite side to that of the hypotenuse.

**Tangent ratio:** the ratio of the length of the opposite side to that of the adjacent side.

**Inverse:** function that has the opposite effect

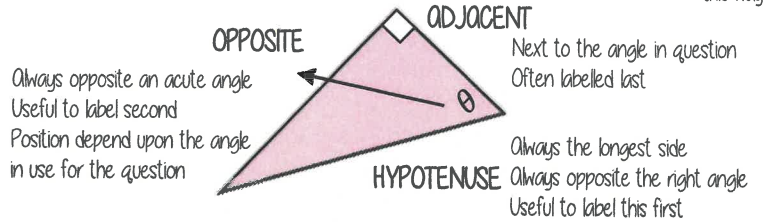
**Hypotenuse:** longest side of a right-angled triangle. It is the side opposite the right-angle.

## Ratio in right-angled triangles



## Hypotenuse, adjacent and opposite

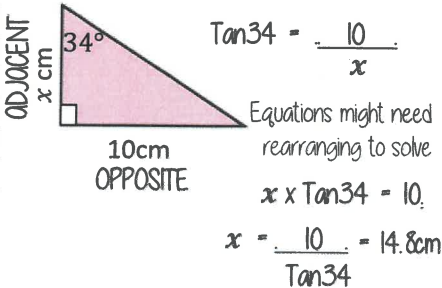
ONLY right-angled triangles are labelled in this way



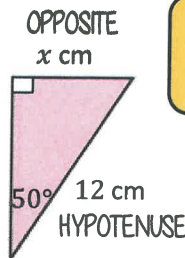
## Tangent ratio: side lengths

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Substitute the values into the tangent formula

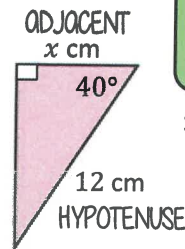


## Sin and Cos ratio: side lengths



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$$

**NOTE**  
The  $\sin(x)$  ratio is the same as the  $\cos(90-x)$  ratio

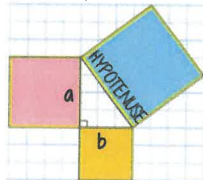


$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

Substitute the values into the ratio formula  
Equations might need rearranging to solve

## Pythagoras theorem

$$\text{Hypotenuse}^2 = a^2 + b^2$$



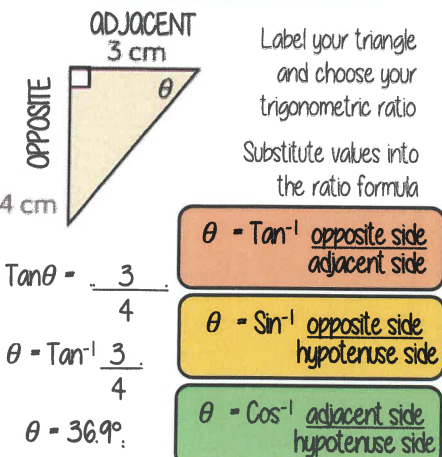
This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras

- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

## Sin, Cos, Tan: Angles

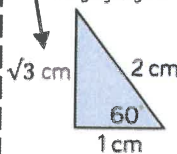
### Inverse trigonometric functions



## Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements

This side could be calculated using Pythagoras



$$\tan 30 = \frac{1}{\sqrt{3}}$$

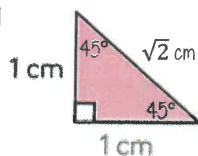
$$\tan 60 = \sqrt{3}$$

$$\cos 30 = \frac{\sqrt{3}}{2}$$

$$\cos 60 = \frac{1}{2}$$

$$\sin 30 = \frac{1}{2}$$

$$\sin 60 = \frac{\sqrt{3}}{2}$$



$$\tan 45 = 1$$

$$\cos 45 = \frac{1}{\sqrt{2}}$$

$$\sin 45 = \frac{1}{\sqrt{2}}$$

## Key angles 0° and 90°

$$\tan 0 = 0$$

$$\tan 90$$

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$$\sin 0 = 0$$

$$\sin 90 = 1$$

$$\cos 0 = 1$$

$$\cos 90 = 0$$



## Trigonometry with bounds

The upper bound of a multiplication is always the two upper bounds multiplied together

The lower bound of a multiplication is always the two lower bounds multiplied together

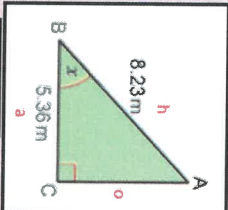
The upper bound of a fraction is always the Upper bound of the numerator

The lower bound of a fraction is always the Lower bound of the numerator

The upper bound of a fraction is always the Upper bound of the denominator

The lower bound of a fraction is always the Lower bound of the denominator

In this diagram, the measurements are correct to 3 significant figures.  
 a Find the upper and lower bounds for the value of  $x$ , to 3 decimal places.  
 b Give the value of  $x$  to a suitable level of accuracy.



- Steps:**
- 1) Find the upper bound and lower bound of the sides
  - 2) Find the upper and lower Value of  $x$  using trigonometry
  - 3) Round both values to the Nearest degree to find a Good estimate for  $x$

UB of  $h = 8.235$   
 LB of  $h = 8.225$   
 UB of  $a = 5.365$   
 LB of  $a = 5.355$

$\cos x_{ub} = \frac{5.365}{8.225}$   
 $\cos x_{lb} = \frac{5.355}{8.235}$

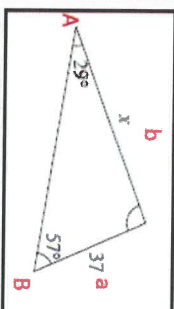
$x_{ub} = \cos^{-1} \frac{5.365}{8.225} = 49.438^\circ$   
 $x_{lb} = \cos^{-1} \frac{5.355}{8.235} = 49.286^\circ$

$x = 49^\circ$  (to the nearest degree)

## Finding a missing side

- Step 1: Label your sides and angles
- Step 2: Substitute known values into the formula
- Step 3: Rearrange the formula to find the missing side

Find the length of side  $x$ .



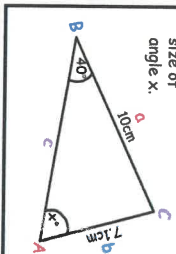
$\frac{37}{\sin 29} = \frac{x}{\sin 57}$   
 $37 \times \sin 57 = x \sin 29$   
 $x = \frac{37 \times \sin 57}{\sin 29}$   
 $x = 64.004^\circ$   
 $x = 64^\circ$

## Sine Rule

The Sine and Cosine Rules are used for finding missing sides and angles on non right angled triangles.

### Finding a missing angle

Find the size of angle  $x$ .



The formula for the sine rule is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{10} = \frac{\sin 40}{7.1}$$

$$\sin A = \frac{\sin 40 \times 10}{7.1}$$

$$A = \sin^{-1} \frac{\sin 40 \times 10}{7.1}$$

$$A = 64.9^\circ$$

## Unit 13b:

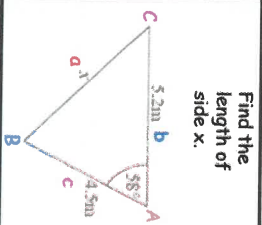
### Further

## Trigonometry

### Cosine Rule: Finding a missing side

Can be used to find missing sides or angles. MUST have 2 sides and 1 angle

Formula for missing side  
 $a^2 = b^2 + c^2 - 2bc \cos A$



Find the length of side  $x$ .

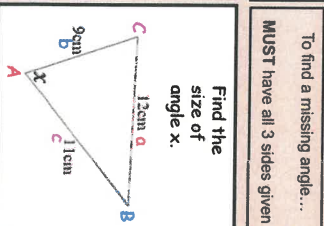
$x^2 = 5.2^2 + 4.5^2 - (2 \times 5.2 \times 4.5 \times \cos 55)$   
 $x^2 = 5.2^2 + 4.5^2 - (24.4)$   
 $x^2 = 27.04 + 20.25 - (24.4)$   
 $x^2 = 47.29 - (24.4)$   
 $x^2 = 22.89$   
 $x = 4.74m$

### Cosine Rule: Finding a missing angle

Formula for missing angle

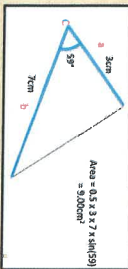
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$\cos x = \frac{9^2 + 11^2 - 12^2}{2 \times 9 \times 11}$   
 $\cos x = \frac{58}{198}$   
 $x = \cos^{-1} \frac{58}{198}$   
 $x = 72.97^\circ$



To find a missing angle... MUST have all 3 sides given

## Cosine Rule



Area =  $0.5 \times 3 \times 7 \times \sin 39 = 9.00cm^2$

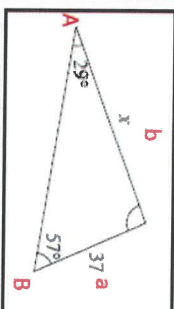
Area =  $\frac{1}{2} a b \sin(C)$

Where C is the angle wedged between two sides a and b.

## Finding a missing side

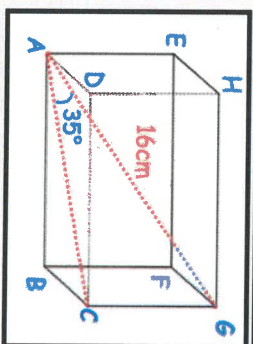
- Step 1: Label your sides and angles
- Step 2: Substitute known values into the formula
- Step 3: Rearrange the formula to find the missing side

Find the length of side  $x$ .



$\frac{37}{\sin 29} = \frac{x}{\sin 57}$   
 $37 \times \sin 57 = x \sin 29$   
 $x = \frac{37 \times \sin 57}{\sin 29}$   
 $x = 64.004^\circ$   
 $x = 64^\circ$

Length AG = 16cm  
 Angle CAG is  $35^\circ$   
 Work out the length of EG.



$$\cos(CAG) = \frac{u}{h}$$

$$\cos(35) = \frac{EG}{16}$$

$$16 \times \cos(35) = EG$$

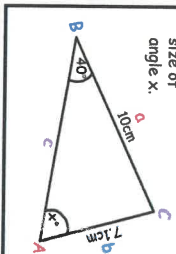
$$EG = 13.1cm$$

## Sine Rule

The Sine and Cosine Rules are used for finding missing sides and angles on non right angled triangles.

### Finding a missing angle

Find the size of angle  $x$ .



The formula for the sine rule is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

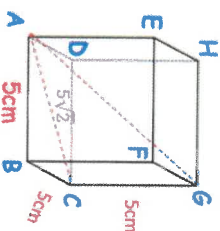
$$\frac{\sin A}{10} = \frac{\sin 40}{7.1}$$

$$\sin A = \frac{\sin 40 \times 10}{7.1}$$

$$A = \sin^{-1} \frac{\sin 40 \times 10}{7.1}$$

$$A = 64.9^\circ$$

1. Shown is a cube with side length 5cm.



Calculate angle CAG.

$$AC^2 = \sqrt{5^2 + 5^2}$$

$$AC^2 = \sqrt{50}$$

$$AC = 5\sqrt{2}$$

$$\tan(CAG) = \frac{5}{5\sqrt{2}}$$

$$CAG = \tan^{-1} \frac{5}{5\sqrt{2}}$$

$$CAG = 35.3^\circ$$

## Trigonometry in 3D



## Sampling

When you are investigating a hypothesis, the population is the whole group that you are interested in.

A population that could possibly be involved in an investigation

A population may divide into groups such as age range. These groups are called strata. In a stratified sample, the number of people taken from each group is proportional to the group size.

### Stratified Sampling

The general frequency table shows information about the population as a whole, and the stratified frequency table shows information about the population as a sample.

| Weight (kg)  | Frequency |
|--------------|-----------|
| 50 <= W < 55 | 7         |
| 55 <= W < 60 | 2         |
| 60 <= W < 65 | 1         |
| 65 <= W < 70 | 1         |
| 70 <= W < 75 | 1         |
| 75 <= W < 80 | 1         |
| 80 <= W < 85 | 2         |
| <b>Total</b> | <b>20</b> |

There are 200 students in Year 11.  
You select an estimate for the number of students in Year 11 whose weight is between 50 kg and 60 kg.  
 $\frac{7}{20} \times 200 = 70$

## Capture Recapture

Capture-Recapture is a technique that can be used to estimate the total population.

$$\frac{M}{N} = \frac{R}{T}$$

M = total marked  
N = total population  
R = number "recaptured"  
T = total captured on 2nd catch

Give wants to estimate the number of bees in a beehive. She catches 50 bees from the beehive.

He marks each bee with a dye. He then lets the bees go.

The next day, Give catches 40 bees from the beehive. 8 of these bees have been marked with the dye.

Work out an estimate for the number of bees in the beehive.

$$M = 50$$

$$T = 40$$

$$R = 8$$

$$N = ?$$

$$\frac{50}{N} = \frac{8}{40}$$

$$50 = \frac{8N}{40}$$

$$2000 = 8N$$

$$250 = N$$

## A Cumulative Frequency table shows a running total of the frequencies

| Mark  | Frequency | Cumulative Frequency |
|-------|-----------|----------------------|
| 1-10  | 3         | 3                    |
| 11-20 | 6         | 3 + 6 = 9            |
| 21-30 | 11        | 9 + 11 = 20          |
| 31-40 | 13        | 20 + 13 = 33         |
| 41-50 | 18        | 33 + 18 = 51         |

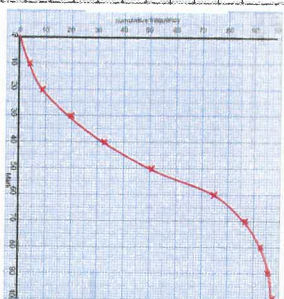
A cumulative frequency table shows how many data values are less than or equal to the upper class boundary of each data class.

The upper class boundary is the highest possible value in each class.

## Cumulative Frequency

### Drawing a Cumulative Frequency Graph

| Mark   | Frequency | Cumulative Frequency |
|--------|-----------|----------------------|
| 1-10   | 3         | 3                    |
| 11-20  | 6         | 9                    |
| 21-30  | 11        | 20                   |
| 31-40  | 13        | 33                   |
| 41-50  | 18        | 51                   |
| 51-60  | 24        | 75                   |
| 61-70  | 12        | 87                   |
| 71-80  | 6         | 93                   |
| 81-90  | 3         | 96                   |
| 91-100 | 2         | 98                   |



Steps:

- 1) Start from 0
- 2) Plot using end points
- 3) Join using a smooth curve

## Unit 14: Further Statistics

### Averages

1 1 3 5 7 9 11 14 16 19 19 20 21

n = 13

$$\text{Median} = \frac{13+1}{2} = \frac{14}{2} = 7\text{th value} = 11$$

$$LQ = \frac{13+1}{4} = \frac{14}{4} = 3.5\text{th value} = 4$$

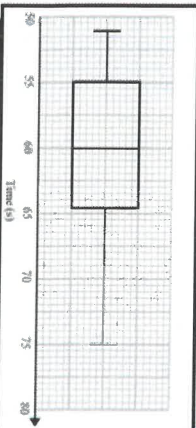
$$UQ = 3.5 \times 3 = 10.5\text{th value} = 19$$

### Drawing a box plot

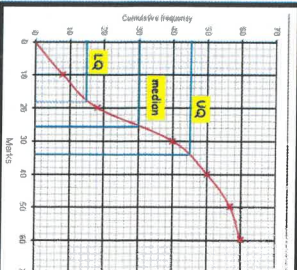
These data, an example of 25 students' running times over one recorded lesson.

|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 42 | 64 | 34 | 55 | 38 | 56 | 59 | 60 | 60 | 61 | 61 | 64 | 67 | 70 | 75 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

Draw a box plot for this information.



## Box Plots + CF Diagrams



**Median** - Middle value found at total frequency  $\div 2$

**Lower quartile** - Value which is a quarter of the way through the data found at total frequency  $\div 4$

**Upper quartile** - Value which is three quarters of the way through the data found at total frequency  $\div 4 \times 3$

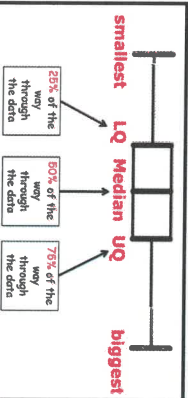
**Interquartile range** = Upper quartile - Lower quartile

**Median** = 26

**Lower Quartile** = 18

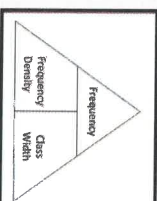
**Upper Quartile** = 34

**IQR** = 34 - 18 = 16

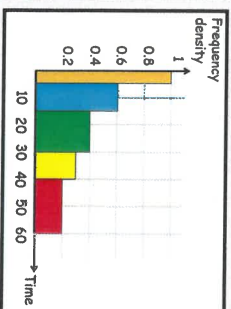


### Drawing a histogram

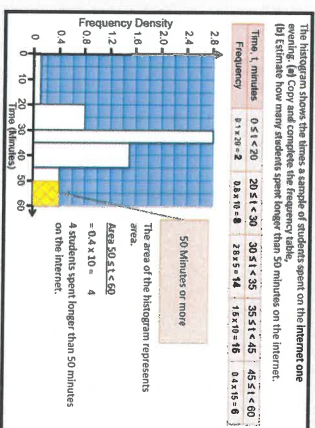
| Time, t, in minutes | Frequency | Frequency density | Frequency |
|---------------------|-----------|-------------------|-----------|
| 0.5 t < 5           | 1         | 5                 | 5         |
| 5.5 t < 15          | 6         | 0.6               | 6         |
| 15.5 t < 30         | 6         | 0.4               | 6         |
| 30.5 t < 40         | 3         | 0.3               | 3         |
| 40.5 t < 60         | 4         | 0.2               | 4         |



This is the formula triangle for a histogram



### Interpreting a Histogram



## Histograms



## Vector Basics

What is a vector?

A vector describes **direction and length**

The magnitude of a vector is its size

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

X = number of moves to the right or left

Y = number of moves up or down

This vector can be written in 3 ways

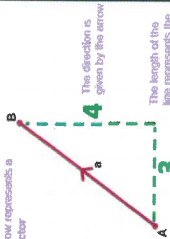
$$a \quad \vec{a} \quad \vec{AB}$$

$$a = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \vec{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\vec{EF} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$$

$$x = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$$

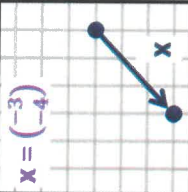
For example this arrow represents a vector



The direction is given by the arrow

The length of the line represents the magnitude

$$\vec{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$



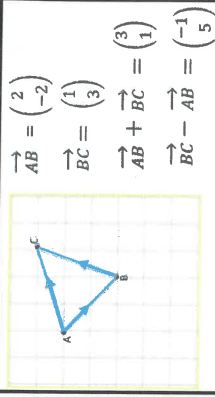
## Vector Arithmetic

If we add two or more vectors together we get a resultant vector

A resultant vector is the vector sum of two or more vectors

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$



$$\vec{AB} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\vec{BC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\vec{AB} + \vec{BC} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\vec{BC} - \vec{AB} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

What's another way of saying  $b + b + b$ ?  $3b$  Vector Scalar

Multiplying a Vector by a Scalar

$$z = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$3z = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

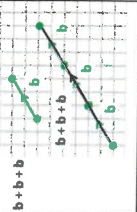
To multiply a Vector by a Scalar, Write the Vector as a Column Vector then multiply each entry in the Column Vector by the Scalar

$$3z = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \times 3 = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

We can multiply a vector by a scalar

A scalar is a quantity that has size but no direction

Vectors that have been multiplied by a scalar are parallel



# Unit 18: Vectors

## Midpoints of Vectors

3. P is the point (1,5), Q is the point (9,3)

a) Write down the vector  $\vec{PQ}$   
Write your answer as a column vector

M is the midpoint of PQ

$$\vec{PM} = \frac{1}{2}\vec{PQ} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$$

## Vectors with ratio

$$\vec{AL} : \vec{LB} = 2 : 1$$

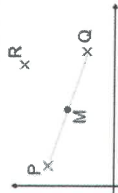
What is:

$$\vec{AB} = a$$

$$\vec{AL} = \frac{2}{3}a$$

$$\vec{LB} = \frac{1}{3}a$$

Diagram NOT accurately drawn



## Vectors in quadrilaterals



OACB is a parallelogram

$$\vec{OA} = a \text{ and } \vec{OB} = b$$

Find i)  $\vec{OC}$  ii)  $\vec{BA}$  iii)  $\vec{CA}$

In terms of a and b



$$\vec{OC} = b + a$$



$$\vec{BA} = a - b$$



$$\vec{CA} = -b$$

$$\vec{CA} = -b - b + b$$

$$\vec{CA} = -b$$

X is a point such that  $\vec{AX} : \vec{XB} = 1 : 4$



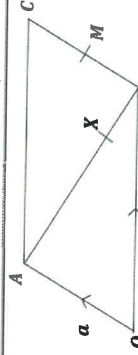
$$a. \vec{AB} = -a + b$$

$$b. \vec{AX} = -\frac{1}{5}a + \frac{4}{5}b$$

$$c. \vec{OX} = \frac{4}{5}a + \frac{1}{5}b$$

$$d. \vec{BX} = \frac{4}{5}a - \frac{4}{5}b$$

## How to show two vectors are parallel



X is a point on AB such that  $\vec{AX} : \vec{XB} = 3 : 1$ . M is the midpoint of BC. Show that  $\vec{XM}$  is parallel to  $\vec{AC}$ .

For any proof question always find the vectors involved first, in this case  $\vec{XM}$  and  $\vec{AC}$ .

The line is to factor out a scalar such that we see the same vector.

The magic words here are "a multiple of".

$$\vec{OC} = a + b$$

$$\vec{XM} = \frac{3}{4}(-a + b) + \frac{1}{2}a = \frac{1}{4}a + \frac{3}{4}b$$

$$= \frac{1}{4}(a + b)$$

$\vec{XM}$  is a multiple of  $\vec{OC}$ , parallel.

a) Find  $\vec{XB}$  in terms of a and b.

$$-2a + 3b$$

b) P is the point on AB such that  $\vec{AP} : \vec{PB} = 2 : 3$ . Show that  $\vec{OP}$  is parallel to the vector  $a + b$ .

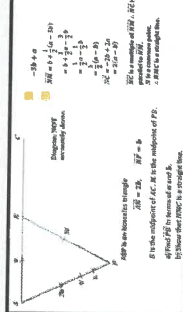
$\vec{OP}$  for  $2a = \frac{2}{5}(3b - 2a) - 2a = \frac{2}{5}(3a - 3b)$   
 $\vec{AP} : \vec{PB} = 2 : 3$   
 Show that  $\vec{OP}$  is parallel to the vector  $a + b$ .

AI for  $\frac{2}{5}a = \frac{2}{5}b$  or  
 AI for  $\frac{2}{5}(a - b)$  is parallel to  $a - b$  as

## Collinear Points

Points A, B and C form a straight line if:  $\vec{AB}$  and  $\vec{BC}$  are parallel (and B is a common point).

Alternatively, we could show  $\vec{AB}$  and  $\vec{AC}$  are parallel. This tends to be easier.



## Vector Proof



# Algebraic Fractions

When we multiply fractions we multiply the numerators together and the denominators together.

$$\frac{1}{1} \times \frac{5}{1} = \frac{1 \times 5}{1 \times 1} = \frac{5}{1} = 5$$

$$\frac{a}{1} \times \frac{2}{1} = \frac{a \times 2}{1 \times 1} = \frac{2a}{1} = 2a$$

$$\frac{3(c-1)}{4} \times \frac{2(c-1)}{4} = \frac{3 \times 2 \times (c-1) \times (c-1)}{4 \times 4} = \frac{6(c-1)^2}{16}$$

When we divide fractions we find the reciprocal of the second fraction and then multiply the fractions together.

$$\frac{6x}{2y} \div \frac{4y}{5} = \frac{6x}{2y} \times \frac{5}{4y} = \frac{30x}{8y^2} = \frac{15x}{4y^2}$$

When we add or subtract fractions we need to have the same denominator in each fraction.

To do this we need to find the lowest common denominator.

$$\frac{x}{3} + \frac{2x+1}{3} = \frac{x + 2x + 1}{3} = \frac{3x + 1}{3}$$

- Step 1: Find the lowest common denominator
- Step 2: Cross multiply
- Step 3: Simplify

Solve  $\frac{x+5}{7} = 5$

$$\begin{aligned} x+5 &= 5 \times 7 \\ x+5 &= 35 \\ x &= 35-5 \\ x &= 30 \end{aligned}$$

Solve the equation  $\frac{x+4}{2} = \frac{x+10}{3}$

$$\begin{aligned} 3(x+4) &= 2(x+10) \\ 3x+12 &= 2x+20 \\ 3x-2x &= 20-12 \\ x &= 8 \end{aligned}$$

- Step 1: Find the lowest common denominator
- Step 2: Cross Multiply
- Step 3: Simplify the numerator
- Step 4: Solve

Solve  $\frac{2x+1}{3} - \frac{x-5}{2} = 4$

$$\begin{aligned} \frac{2(2x+1) - 3(x-5)}{3 \times 2} &= 4 \\ \frac{4x+4+3x-15}{6} &= 4 \\ \frac{7x-11}{6} &= 4 \\ 7x-11 &= 4 \times 6 \\ 7x-11 &= 24 \\ 7x &= 24+11 \\ 7x &= 35 \\ x &= 5 \end{aligned}$$

# Changing the subject

- Circle the subject that we want to isolate.
- Identify what needs to be eliminated.
- Inverse the operation, and apply to both sides.
- Inverse the operation, and apply both sides.

Make x the subject of:

$$\begin{aligned} ax + b &= cx + d \\ ax - cx &= d - b \\ x(a - c) &= d - b \\ x &= \frac{d - b}{a - c} \end{aligned}$$

Make c the subject

$$\begin{aligned} cm + ca &= z \\ c(m + a) &= z \\ m + a &= \frac{z}{c} \\ c &= \frac{z}{m + a} \end{aligned}$$

# Simplifying Surds

- Find two factors of the number (one of them needs to be a square number)
- Check the surd in half
- Simplify

$$\frac{\sqrt{25}}{4} = \frac{\sqrt{5 \times 5}}{4} = \frac{5}{4}$$

Simplify  $\sqrt{2} + \sqrt{8}$

$$\begin{aligned} &= \sqrt{2} + \sqrt{4 \times 2} \\ &= \sqrt{2} + 2\sqrt{2} \\ &= 3\sqrt{2} \end{aligned}$$

# Surds

Multiplying Surds

$$\sqrt{5} \times \sqrt{6} = \sqrt{30}$$

$$\begin{aligned} (2 + \sqrt{3})(2 + \sqrt{3}) &= 4 + 2\sqrt{3} + 2\sqrt{3} + \sqrt{3} \times \sqrt{3} \\ &= 4 + 4\sqrt{3} + 3 \\ &= 7 + 4\sqrt{3} \end{aligned}$$

# Rationalising the denominator

Rationalise the denominator for

$$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

Rationalise the denominator for

$$\frac{2}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{2(\sqrt{3} - 1)}{3 - 1} = \frac{2\sqrt{3} - 2}{2} = \sqrt{3} - 1$$

# Unit 17: Further Algebra

Prove that  $(3n+1)^2 - (3n-1)^2$  is a multiple of 4 for all positive integer values of n

Before you begin, highlight the key words and information from the question

Prove that  $(3n+1)^2 - (3n-1)^2$  is a multiple of 4 for all positive integer values of n

STEP 1: EXPAND

$$\begin{aligned} (3n+1)^2 - (3n-1)^2 &= (9n^2 + 6n + 1) - (9n^2 - 6n + 1) \\ &= 9n^2 + 6n + 1 - 9n^2 + 6n - 1 \\ &= 12n \end{aligned}$$

STEP 2: SIMPLIFY

$$12n = 4 \times 3n$$

STEP 3: FACTORISE

$$12n = 4 \times 3n$$

STEP 3: JUSTIFY

$4 \times 3n \rightarrow$  always divisible by 4 and hence a multiple of 4

Prove that  $(n+1)^2 - (n-1)^2 + 1$  is always odd for all positive integer values of n

Expand

$$(n+1)^2 - (n-1)^2 + 1 = (n^2 + 2n + 1) - (n^2 - 2n + 1) + 1$$

Simplify

$$= n^2 + 2n + 1 - n^2 + 2n - 1 + 1 = 4n + 1$$

Justify

$4 \times n$  is always even. Any even number add 1 is odd

PROVE THAT THE SUM OF TWO CONSECUTIVE NUMBERS IS ALWAYS AN ODD NUMBER

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# Functions

A function is something which provides a rule on how to map inputs to outputs.

From primary school you might have seen this as a 'function machine'.



Input  $x$  goes into a box labeled  $f$ , and  $2x$  comes out. Below the box is  $f(x) = 2x$ .

Output  $2x$

Input  $x$

Name of the function (usually  $f$  or  $g$ )

$f(x) = 2x$

You are given that  $f(x) = 3x + 1$

Find  $f(1)$

This means we need to substitute  $x$  for 1

$$f(1) = 3(1) + 1 = 3 + 1 = 4$$

If  $p = 3$  and  $x = 2$

Write  $p$  in terms of  $y$

$$p = 3x$$

$$p = 3y$$

$$p = 3y$$

$$f(x) = x^2 + 2$$

If  $f(a) = 38$ , what is  $a$ ?

$$a^2 + 2 = 38$$

$$a^2 = 36$$

$$a = \pm 6$$

Anything inside the brackets applies just to that  $x$

Anything outside the brackets applies just to the whole function

# Inverse Functions

$f(x) = 2x + 1$

find  $f^{-1}(x)$

$$y = 2x + 1$$

$$y - 1 = 2x$$

$$x = \frac{y-1}{2}$$

$$f^{-1}(x) = \frac{x-1}{2}$$

STEP 1: Write the output  $f(x)$  as  $y$

STEP 2: Get the input in terms of the output (make  $x$  the subject)

STEP 3: Swap  $y$  back for  $x$  and  $x$  back for  $f^{-1}(x)$

# Composite Functions

Given  $f(x) = 2x$

$g(x) = x + 1$

find  $fg(1)$

$$g(1) = 1 + 1 = 2$$

$$f(2) = 2 \times 2 = 4$$

$$fg(1) = 4$$

$$f(g(1)) = 4$$

$$g(f(1)) = 2 + 1 = 3$$

$$f(3) = 2 \times 3 = 6$$

$$fg(1) = 4$$

$$f(g(1)) = 4$$

$$g(f(1)) = 3$$

$$f(3) = 6$$

# Functions

# Algebraic Proof

An identity in maths is represented by the  $\equiv$  symbol

An identity is an equation which is always true no matter which values are chosen

$$(x+1)^2 \equiv x^2 + 2x + 1$$

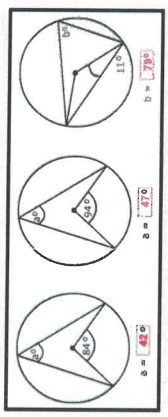
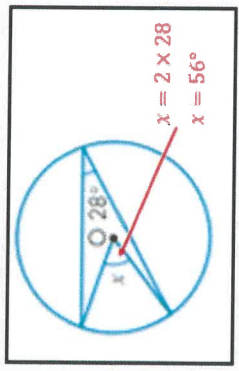
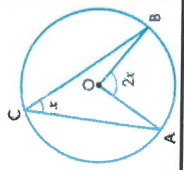


# Unit 16: Circle Theorems

## Circle Theorem 1:

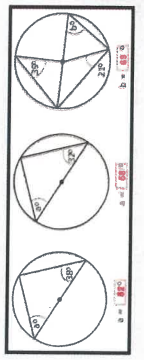
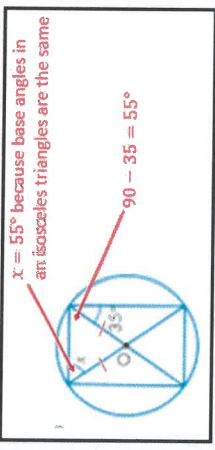
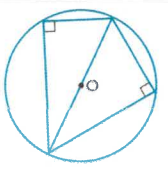
"The angle at the centre of a circle is twice the angle at the circumference."

Angle  $AOB = 2 \times ACB$



## Circle Theorem 2:

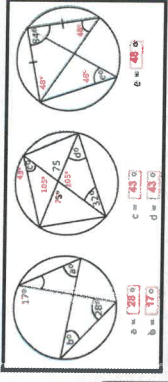
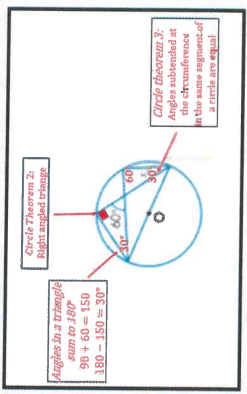
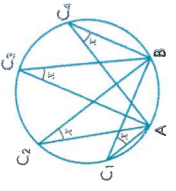
"Every angle at the circumference of a semicircle that is subtended by the diameter of the semicircle is a right-angle."



## Circle Theorem 3:

"Angles subtended at the circumference in the same segment of a circle are equal."

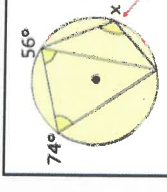
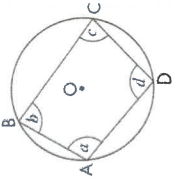
Points  $C_1, C_2, C_3$  and  $C_4$  on the circumference are subtended by the same arc, AB.



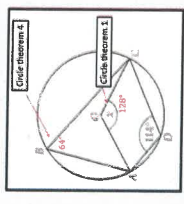
## Circle Theorem 4:

"The sum of the opposite angles in a cyclic quadrilateral is  $180^\circ$ ."

$a + c = 180^\circ$  and  $b + d = 180^\circ$

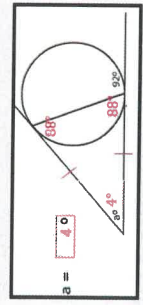
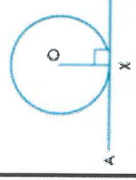


Circle theorem 4:  
Opposite angles in a cyclic quadrilateral  
Sum to  $180^\circ$   
 $180 - 74 = 106$   
 $x = 106^\circ$



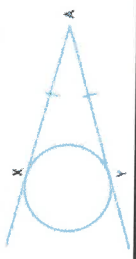
## Circle Theorem 5:

"When a radius meets a tangent, it always makes a  $90^\circ$  angle."



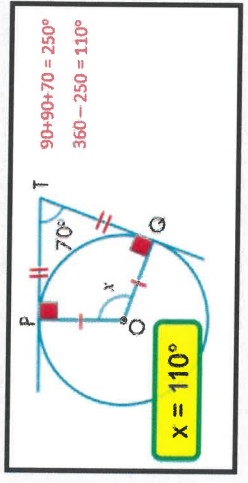
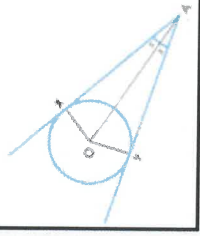
## Circle Theorem 6:

"Tangents to a circle from an external point to the points of contact are equal in length."



## Circle Theorem 7:

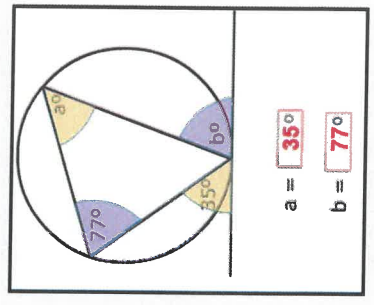
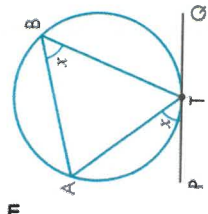
"The line joining an external point to the centre of the circle bisects the angle between the tangents."



## Circle Theorem 9:

The Alternate Segment Theorem

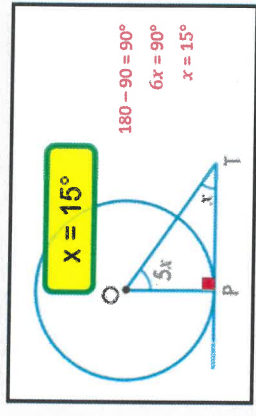
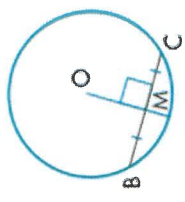
"The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment."



## Circle Theorem 8:

"A radius bisects a chord at  $90^\circ$ ."

If O is the centre of the circle, angle  $BMO = 90^\circ$  and  $BM = CM$ .



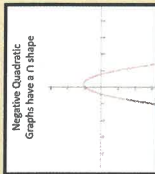


# Plotting Quadratic Graphs

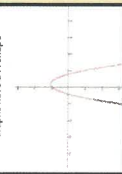
## Quadratic Graphs

Quadratic graphs are curved and symmetrical

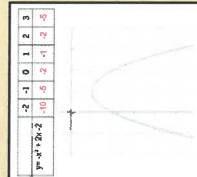
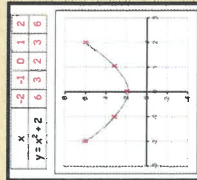
Positive Quadratic  
Graphs have a U shape



Negative Quadratic  
Graphs have an inverted U shape



When you square a negative number the answer is positive



## Finding the roots of a Quadratic Graph

To find the roots of a graph we factorise

Find the roots for this equation

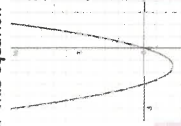
$$x^2 + 3x = 0$$

$$x(x + 3) = 0$$

$$x = 0 \text{ or } x = -3$$

Steps:

- 1) Set the equation equal to 0
- 2) Factorise
- 3) Solve for x



## Finding the turning points of a Quadratic Graph

Find the turning point for the equation

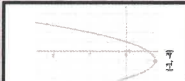
$$y = x^2 + 2x - 3$$

Will the graph have a minimum or a maximum?

Minimum

What are the coordinates of the turning point?

$$(x + 1)^2 = 4$$



To find a maximum or minimum point you complete the square

## Solving simultaneous equations by using the graph

The diagram shows the graphs of

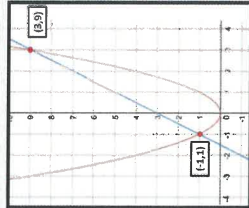
$$y = x^2$$

$$y = 2x + 3$$

Use the diagram to solve this pair of simultaneous equations:

$$y = x^2$$

$$y = 2x + 3$$



The points where the graphs intersect are the solutions of the simultaneous equations.

$$x = -1, y = 1 \text{ or } x = 3, y = 9$$

## Simultaneous Equations

Solving simultaneous equations by plotting the graphs

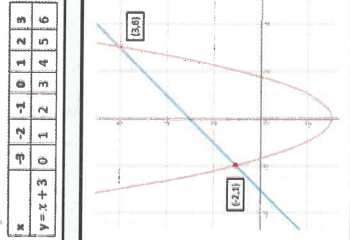
Sketch the graph of  $y = x^2 - 3$

|                        |    |    |    |    |    |   |   |
|------------------------|----|----|----|----|----|---|---|
| x                      | -3 | -2 | -1 | 0  | 1  | 2 | 3 |
| y = x <sup>2</sup> - 3 | 6  | 1  | -2 | -3 | -2 | 1 | 6 |

By drawing a suitable line on your graph, solve this pair of simultaneous equations:

$$y = x^2 - 3$$

$$y = x + 3$$



$$x = -2, y = 1 \text{ or } x = 3, y = 6$$

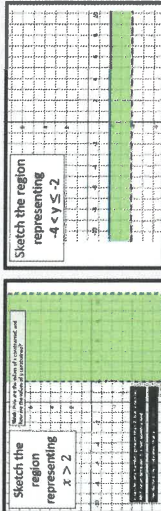
The points where the graphs intersect are the solutions of the simultaneous equations.

Use a table of values to help you plot the graphs more accurately

# Unit 15: Equations and Graphs

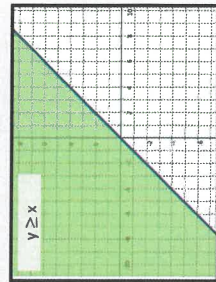
## Representing graphical inequalities

Sketch the region representing  $x > 2$



If the line is a boundary for values that are included, the line must be drawn with a **solid line**

If the line is a boundary for values that are **not** included, the line must be drawn with a **dashed line**



## Graphical Inequality regions

## Solving graphical inequalities

On the grid, shade the region which satisfy the inequalities:

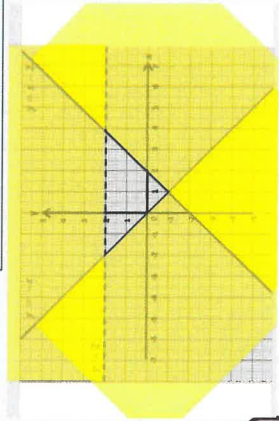
$$y \geq -x$$

$$y < 2$$

$$y \geq x - 2$$

To solve inequalities graphically

1. Draw the lines for the inequalities (treating them as equations (remember solid or dashed lines))
2. Choose a point on either side of the line to test if the inequality is true or not
3. Shade the region that satisfies each inequality
4. The solution will be the unshaded region

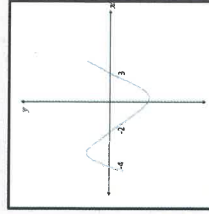


## Sketching cubic graphs

Sketch the cubic graph

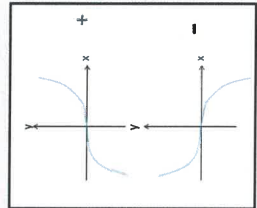
$$(x + 2)(x - 3)(x + 4)$$

Set each bracket to 0 this will give you the roots



Positive cubic graph

Negative cubic graph



## Finding roots of cubic graphs by iteration

a) Show that the equation  $x^3 - x - 19 = 0$  can be arranged to give  $x = \sqrt[3]{x + 19}$

$$x^3 = x + 19$$

$$x = \sqrt[3]{x + 19}$$

b) Starting with  $x_0 = 0$ , use the iteration formula  $x_{n+1} = \sqrt[3]{x_n + 19}$  four times to find an approximate solution to  $x^3 - x - 19 = 0$ . Write all the digits on your calculator display.

$$x_0 = 0$$

$$x_1 = 2.6680401 \dots$$

$$x_2 = 2.7878899 \dots$$

$$x_3 = 2.7930050 \dots$$

$$x_4 = 2.7932236 \dots$$

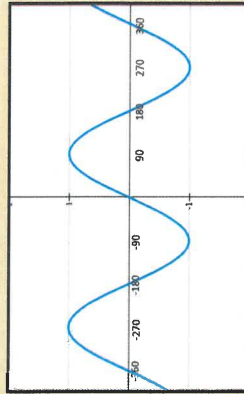
$$x \approx 2.79$$

## Cubic Graphs



## The graphs of sine, cos and Tan

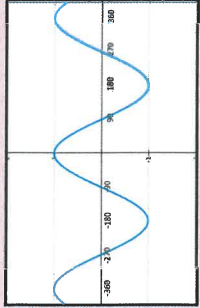
### Sine Graph



The Sine Graph repeats every  $360^\circ$   
It's largest value is 1  
It's smallest value is -1

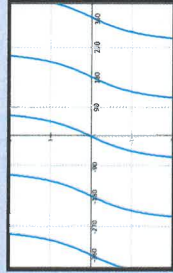
Each graph is shown between the range of  $-360$  degrees and  $+360$  degrees. The range can be changed/restricted so make sure you read the range carefully in a question.

### Cos Graph



The Cosine Graph repeats every  $360^\circ$   
It's largest value is 1  
It's smallest value is -1

### Tan Graph



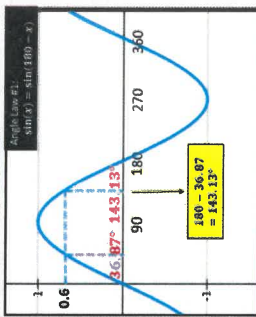
The Tan Graph repeats every  $180^\circ$   
There are asymptotes (the dashed lines) that the tan graph never touches.  
It doesn't have a maximum or minimum point, it goes between negative infinity and positive infinity.

### Steps

- 1) Sketch the trig graph within the specified range.
- 2) Draw a straight horizontal line at your given value notice how many times it crosses the graph (this is how many answers there are)
- 3) Substitute the value given into the inverse trig function into your calculator.
- 4) Round to a suitable degree of accuracy
- 5) This gives you one answer
- 6) Use your answer to find other solutions of the equation.

Solve  $\sin(x) = 0.6$  in the range  $0 \leq x < 360$

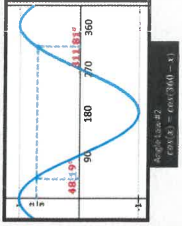
- 1) Draw a straight horizontal line at 0.6 (you can see we will have 2 solutions)
- 2) Substitute 0.6 into the inverse trig function into your calculator
- 3) Round to a suitable degree of accuracy:  $36.87^\circ$
- 4) Subtract  $36.87^\circ$  from  $180^\circ$  to find the second solution.



$$x = \sin^{-1}(0.6) = 36.87^\circ, 143.13^\circ$$

## Solving Equations with Trig Graphs

Solve  $3\cos(x) = 2$  in the range  $0 \leq x < 360$



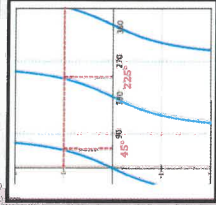
- 1) Rearrange first to eliminate the 3
- 2) Draw a horizontal line at  $\frac{2}{3}$
- 3) Substitute  $\frac{2}{3}$  into the inverse cos function
- 4) Round the answer to a suitable degree of accuracy
- 5) Find all solutions

$$x = \cos^{-1}\left(\frac{2}{3}\right) = 48.19^\circ, 311.81^\circ$$

Solve  $\tan(x) = 1$  in the range  $0 \leq x < 360$

$$x = \tan^{-1}(1) = 45^\circ, 225^\circ$$

- 1) Draw a horizontal line at 1
- 2) Substitute 1 into the inverse tan function
- 3) Find all solutions

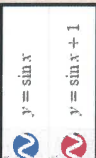


## Unit 13a: Trigonometric Graphs

When we add or subtract outside the bracket it translates the graph in y

$$f(x) + 1 \text{ translates the graph by } \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$f(x) - 1 \text{ translates the graph by } \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$



When we add or subtract inside the bracket it translates the graph in x (in the opposite direction)

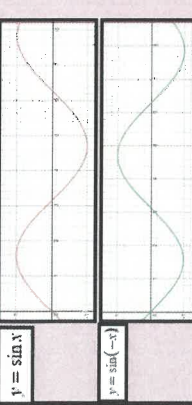
$$f(x + 1) \text{ translates the graph by } \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$f(x - 1) \text{ translates the graph by } \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



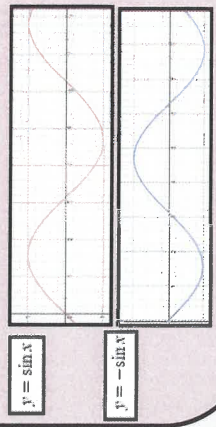
When we have a negative change inside the bracket

$$f(-x) \text{ is a reflection of the graph in the } y - \text{axis}$$



When we have a negative change outside the bracket

$$-f(x) \text{ is a reflection of the graph in the } x - \text{axis}$$

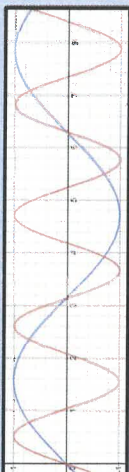


When we have a change inside the bracket

$$f(ax) \text{ squashes on the } x \text{ axis by a factor of } a$$



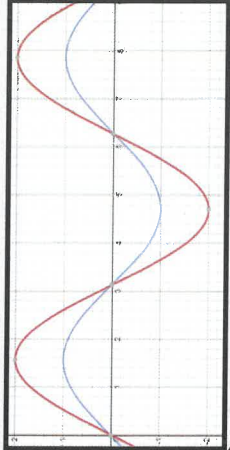
$f(3x)$  → Squash on x-axis by factor of 3



When we have a change outside the bracket

$$af(x) \text{ stretched on the } y - \text{axis by a factor of } a$$

$2f(x)$  → Stretch on y-axis by factor of 2



## Transformations of Graphs